FINAL JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Wednesday 1st February, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

61. $\lim\limits_{n\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+...+\frac{1}{2n}\right)$ is equal to

(1) 0 (2) $\log_e 2$

(3) $\log_e\left(\frac{3}{2}\right)$ (4) $\log_e\left(\frac{2}{3}\right)$

Official Ans. by NTA (2)

Allen Ans. (2)

62. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(1) $(\sim p) \wedge (\sim q)$ (2) $p \wedge (\sim q)$

(3) $(\sim p) \vee (\sim q)$ (4) $(\sim p) \vee q$

Official Ans. by NTA (1)

Allen Ans. (1)

63. In a binomial distribution B(n, p), the sum and product of the mean & variance are 5 and 6 respectively, then find $6(n + p - q)$ is equal to :-

(1) 51

(2) 52

(3) 53

(4) 50

Official Ans. by NTA (2)

Allen Ans. (2)

64. The sum to 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+....$ is:-

(1) $\frac{59}{111}$ (2) $\frac{55}{111}$

(3) $\frac{56}{111}$ (4) $\frac{58}{111}$

Official Ans. by NTA (2)

Allen Ans. (2)

65. The value of

$\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+....+\frac{1}{49!2!}+\frac{1}{51!1!}$ is

(1) $\frac{2^{50}}{50!}$ (2) $\frac{2^{50}}{51!}$

(3) $\frac{2^{51}}{51!}$ (4) $\frac{2^{51}}{50!}$

Official Ans. by NTA (2)

Allen Ans. (2)

66. If the orthocentre of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is

(1) $x^2 - 19x + 90 = 0$

(2) $x^2 - 18x + 80 = 0$

(3) $x^2 - 22x + 120 = 0$

(4) $x^2 - 20x + 99 = 0$

Official Ans. by NTA (4)

Allen Ans. (4)

67. For a triangle ABC, the value of $\cos2A + \cos2B + \cos2C$ is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) Perimeter of ΔABC is $18\sqrt{3}$

(2) $\sin2A + \sin2B + \sin2C = \sin A + \sin B + \sin C$

(3) $\overrightarrow{MA}.\overrightarrow{MB} = -18$

(4) area of ΔABC is $\frac{27\sqrt{3}}{2}$

Official Ans. by NTA (4)

Allen Ans. (4)

68. The combined equation of the two lines ax+by+c=0 and a′x + b′y + c′ = 0 can be written as (ax + by + c) (a′x + b′y + c′) = 0

The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is

(1) $3x^2 + 5xy + 2y^2 = 0$ (2) $x^2 - y^2 + 10xy = 0$

(3) $3x^2 + xy - 2y^2 = 0$ (4) $x^2 - y^2 - 10xy = 0$

Official Ans. by NTA (4)

Allen Ans. (4)

69. The shortest distance between the lines $\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$ and $\frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$ is

(1) $7\sqrt{3}$ (2) $5\sqrt{3}$

(3) $6\sqrt{3}$ (4) $4\sqrt{3}$

Official Ans. by NTA (3)

Allen Ans. (3)

70. Let S denote the set of all real values of λ such that the system of equations

λx + y + z = 1

x + λy + z = 1

x + y + λz = 1

is inconsistent, then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to

(1) 2 (2) 12

(3) 4 (4) 6

Official Ans. by NTA (4)

Allen Ans. (4)

71. Let

$$S = \left\{x : x \in \mathbb{R} \text{ and } \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}.$$

Then n (S) is equal to

(1) 2 (2) 4

(3) 6 (4) 0

Official Ans. by NTA (2)

Allen Ans. (2)

72. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Then $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$ is equal to

(1) 0 (2) $\frac{-2\pi}{3}$

(3) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (4) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Official Ans. by NTA (2)

Allen Ans. (1)

73. If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then 3(α + β + γ) is equal to

(1) 11 (2) 9

(3) 10 (4) 12

Official Ans. by NTA (4)

Allen Ans. (4)

74. If y = y(x) is the solution curve of the differential equation $\frac{dy}{dx} + y \tan x = x \sec x$, $0 \le x \le \frac{\pi}{3}$, y(0) = 1, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(3) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

(4) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Official Ans. by NTA (1)

Allen Ans. (1)

75. Let R be a relation on $\mathbb{R}$, given by R = {(a, b) : 3a – 3b + $\sqrt{7}$ is an irrational number}. Then R is

(1) Reflexive but neither symmetric nor transitive

(2) Reflexive and transitive but not symmetric

(3) Reflexive and symmetric but not transitive

(4) An equivalence relation

Official Ans. by NTA (1)

Allen Ans. (1)

76. Let the image of the point P(2, –1, 3) in the plane x + 2y – z = 0 be Q. Then the distance of the plane 3x + 2y + z + 29 = 0 from the point Q is

(1) $\frac{22\sqrt{2}}{7}$

(2) $\frac{24\sqrt{2}}{7}$

(3) $2\sqrt{14}$

(4) $3\sqrt{14}$

Official Ans. by NTA (4)

Allen Ans. (4)

77. Let $f(x)=\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. If α a β respectively are the maximum and the minimum values of f, then

(1) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$

(2) $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$

(3) $\alpha^2 - \beta^2 = 4\sqrt{3}$

(4) $\alpha^2 + \beta^2 = \frac{9}{2}$

Official Ans. by NTA (1)

Allen Ans. (1)

78. Let $f(x)=2x + \tan^{-1}x$ and $g(x) = \log_e(\sqrt{1+x^2}+x)$, $x \in [0, 3]$. Then

(1) There exists $x \in [0,3]$ such that $f'(x) < g'(x)$

(2) max f(x) > max g(x)

(3) There exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x)$, $\forall\ x \in (x_1, x_2)$

(4) min f′(x) = 1 + max g′ (x)

Official Ans. by NTA (2)

Allen Ans. (2)

79. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is

(1) 1072

(2) 1792

(3) 1216

(4) 1456

Official Ans. by NTA (1)

Allen Ans. (1)

80. The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$, y(1) = 0 is 4π.

Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(1) $2\sqrt{3}$ (2) $\frac{2\sqrt{3}}{3}$

(3) 2 (4) $\frac{4\sqrt{3}}{3}$

Official Ans. by NTA (4)

Allen Ans. (4)

SECTION-B

81. Let $a_1 = 8, a_2, a_3, \ldots a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is _____.

Official Ans. by NTA (754)

Allen Ans. (754)

82. A(2, 6, 2), B(–4, 0, λ), C(2, 3, –1) and D(4, 5, 0), $|\lambda| \le 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5 - 6\lambda$ is equal to _____.

Official Ans. by NTA (11)

Allen Ans. (11)

83. The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7 is ____.

Official Ans. by NTA (514)

Allen Ans. (514)

84. The remainder when $19^{200} + 23^{200}$ is divided by 49, is ____ .

Official Ans. by NTA (29)

Allen Ans. (29)

85. If

$$\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7}\,dx = \frac{1}{l}(11)^{m/n}$$

where l, m, n $\in \mathbb{N}$, m and n are coprime then l + m + n is equal to ____.

Official Ans. by NTA (63)

Allen Ans. (63)

86. If $f(x) = x^2 + g'(1)x + g''(2)$ and

$g(x) = f(1)x^2 + xf'(x) + f''(x)$,

then the value of $f(4) - g(4)$ is equal to _____.

Official Ans. by NTA (14)

Allen Ans. (14)

87. Let $\vec{v} = \alpha\hat{i} + 2j - 3k$, $\vec{w} = 2\alpha\hat{i} + j - k$, and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}.\hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then m + n is equal to _____.

Official Ans. by NTA (3501)

Allen Ans. (3501)

88. The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is _____.

Official Ans. by NTA (50400)

Allen Ans. (50400)

89. Let A be the area bounded by the curve $y = x|x - 3|$, the x-axis and the ordinates x = – 1 and x = 2. Then 12A is equal to _____.

Official Ans. by NTA (62)

Allen Ans. (62)

90. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to _____.

Official Ans. by NTA (1)

Allen Ans. (1)