

FINAL JEE-MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09th April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH SOLUTION

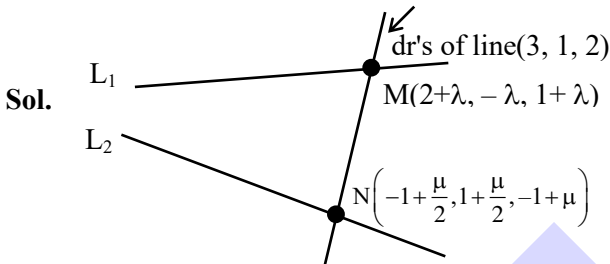
SECTION-A

1. Let the line L intersect the lines
 $x - 2 = -y = z - 1$, $2(x + 1) = 2(y - 1) = z + 1$
 and be parallel to the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$.

Then which of the following points lies on L ?

- (1) $\left(-\frac{1}{3}, 1, 1\right)$ (2) $\left(-\frac{1}{3}, 1, -1\right)$
 (3) $\left(-\frac{1}{3}, -1, -1\right)$ (4) $\left(-\frac{1}{3}, -1, 1\right)$

Ans. (2)



$$L_1 : \frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{1} = \lambda$$

$$L_2 : \frac{x+1}{\frac{1}{2}} = \frac{y-1}{\frac{1}{2}} = \frac{z+1}{1} = \mu$$

dr of line MN will be

$\langle 3 + \lambda - \frac{\mu}{2}, -1 - \lambda - \frac{\mu}{2}, 2 + \lambda - \mu \rangle$ & it will be proportional to $\langle 3, 1, 2 \rangle$

$$\therefore \frac{3 + \lambda - \frac{\mu}{2}}{3} = \frac{-1 - \lambda - \frac{\mu}{2}}{1} = \frac{2 + \lambda - \mu}{2}$$

$$\begin{aligned} \underbrace{3 + \lambda - \frac{\mu}{2}}_{4\lambda + \mu = -6} &= \underbrace{-1 - \lambda - \frac{\mu}{2}}_{4 + 3\lambda = 0} = \frac{2 + \lambda - \mu}{2} \end{aligned}$$

$$\Rightarrow \lambda = -\frac{4}{3} \text{ \& \; } \mu = -\frac{2}{3}$$

\therefore Coordinate of M will be $\left(\frac{2}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

and equation of required line will be.

$$\frac{x - \frac{2}{3}}{3} = \frac{y - \frac{4}{3}}{1} = \frac{z + \frac{1}{3}}{2} = k$$

So any point on this line will be

$$\left(\frac{2}{3} + 3k, \frac{4}{3} + k, -\frac{1}{3} + 2k\right)$$

$$\therefore \frac{2}{3} + 3k = -\frac{1}{3} \Rightarrow k = -\frac{1}{3}$$

\therefore Point lie on the line for

$$k = -\frac{1}{3} \text{ is } \left(-\frac{1}{3}, 1, -1\right)$$

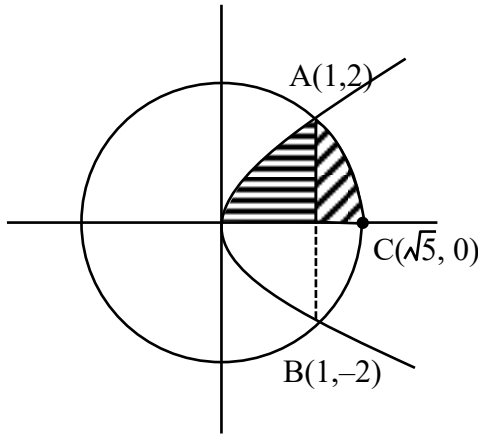
2. The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to :

- (1) $\frac{2}{3} + 5 \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (2) $\frac{1}{3} + 5 \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
 (3) $\frac{1}{3} + \sqrt{5} \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (4) $\frac{2}{3} + \sqrt{5} \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Ans. (1)

Sol. $y^2 = 4x$
 $x^2 + y^2 = 5$

\therefore Area of shaded region as shown in the figure will be



$$A_1 = \int_0^1 \sqrt{4x} \, dx + \int_1^{\sqrt{5}} \sqrt{5-x^2} \, dx$$

$$= \frac{4}{3} \left[x^{\frac{3}{2}} \right]_0^1 + \left[\frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_1^{\sqrt{5}}$$

$$= \frac{1}{3} + \frac{5\pi}{4} - \frac{5}{2} \sin^{-1} \left(\frac{1}{\sqrt{5}} \right)$$

∴ Required Area = 2 A₁

$$= \frac{2}{3} + \frac{5\pi}{2} - 5 \sin^{-1} \left(\frac{1}{\sqrt{5}} \right)$$

$$= \frac{2}{3} + 5 \left(\frac{\pi}{2} - \sin^{-1} \frac{1}{\sqrt{5}} \right)$$

$$= \frac{2}{3} + 5 \cos^{-1} \frac{1}{\sqrt{5}}$$

$$= \frac{2}{3} + 5 \sin^{-1} \left(\frac{2}{\sqrt{5}} \right)$$

3. The solution curve, of the differential equation

$$2y \frac{dy}{dx} + 3 = 5 \frac{dy}{dx}, \text{ passing through the point}$$

(0, 1) is a conic, whose vertex lies on the line :

- (1) $2x + 3y = 9$ (2) $2x + 3y = -9$
 (3) $2x + 3y = -6$ (4) $2x + 3y = 6$

Ans. (1)

Sol. $(2y-5) \frac{dy}{dx} = -3$

$$(2y-5)dy = -3dx$$

$$2 \cdot \frac{y^2}{2} - 5y = -3x + \lambda$$

∴ Curve passes through (0, 1)

$$\Rightarrow \lambda = -4$$

∴ Curve will be

$$\left(y - \frac{5}{2} \right)^2 = -3 \left(x - \frac{3}{4} \right)$$

∴ Vertex of parabola will be $\left(\frac{3}{4}, \frac{5}{2} \right)$

$$\therefore 2x + 3y = 9$$

4. A ray of light coming from the point P (1, 2) gets reflected from the point Q on the x-axis and then passes through the point R (4, 3). If the point S (h, k) is such that PQRS is a parallelogram, then hk^2 is equal to :

- (1) 80 (2) 90
 (3) 60 (4) 70

Ans. (4)

Sol.

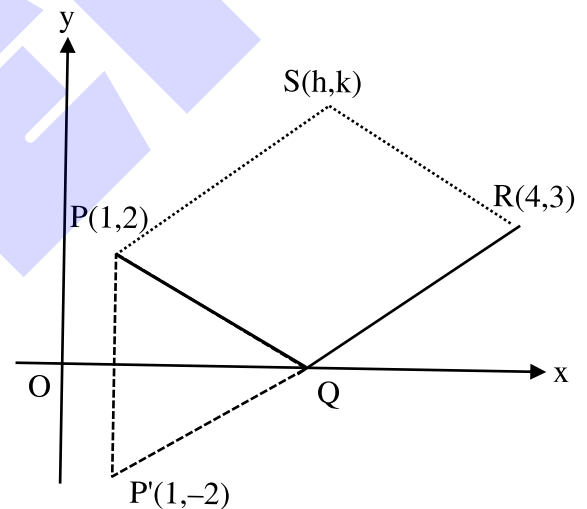


Image of P wrt x-axis will be P'(1, -2) equation of line joining P'R will be

$$y - 3 = \frac{5}{3}(x - 4)$$

Above line will meet x-axis at Q where

$$y = 0 \Rightarrow x = \frac{11}{5}$$

$$\therefore Q \left(\frac{11}{5}, 0 \right)$$

∴ PQRS is parallelogram so their diagonals will bisect each other

$$\Rightarrow \frac{4+1}{2} = \frac{11+h}{5} \& \frac{2+3}{2} = \frac{k+0}{2}$$

$$\Rightarrow h = \frac{14}{5} \& k = 5$$

$$\therefore hk^2 = \frac{14}{5} \times 5^2 = 70$$

5. Let $\lambda, \mu \in \mathbb{R}$. If the system of equations

$$3x + 5y + \lambda z = 3$$

$$7x + 11y - 9z = 2$$

$$97x + 155y - 189z = \mu$$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to :

(1) 25 (2) 24

(3) 27 (4) 22

Ans. (1)

Sol. $3x + 5y + \lambda z = 3$

$$7x + 11y - 9z = 2$$

$$97x + 155y - 189z = \mu$$

$$93x + 155y + 31\lambda z = 31\mu$$

$$97x + 155y - 189z = \mu$$

$$\begin{array}{r} - & - & + & - \\ \hline -4x + (31\lambda + 189)z = 93 - \mu \end{array}$$

$$1085x + 1705y - 1395z = 310$$

$$1067x + 1705y - 2079z = 11\mu$$

$$\begin{array}{r} - & - & + & - \\ \hline 18x + 684z = 310 - 11\mu \end{array}$$

$$-36x + 9(31\lambda + 189)z = 9(93 - \mu)$$

$$36x + 1368z = 2(310 - 11\mu)$$

$$(279\lambda + 3069)z = 1457 - 31\mu$$

for infinite solutions -

$$\lambda = \frac{-3069}{279} = \frac{-341}{31}$$

$$\mu = \frac{1457}{31}$$

$$\mu + 2\lambda = \frac{1457 - 682}{31} = \frac{775}{31} = 25$$

6. The coefficient of x^{70} in $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \dots + x^{54}(1+x)^{46}$ is ${}^{99}C_p - {}^{46}C_q$.

Then a possible value to $p + q$ is :

(1) 55 (2) 61

(3) 68 (4) 83

Ans. (4)

Sol. $x^2(1+x)^{98} + x^3(1+x)^{97} + x^4(1+x)^{96} + \dots$

$$+ \dots + x^{54}(1+x)^{46}$$

$$\text{Coeff. of } x^{70} : {}^{98}C_{68} + {}^{97}C_{67} + {}^{96}C_{66} + \dots$$

$$+ {}^{47}C_{17} + {}^{46}C_{16}$$

$$= {}^{46}C_{30} + {}^{47}C_{30} + \dots + {}^{98}C_{30}$$

$$= ({}^{46}C_{31} + {}^{46}C_{30}) + {}^{47}C_{30} + \dots + {}^{98}C_{30} - {}^{46}C_{31}$$

$$= {}^{47}C_{31} + {}^{47}C_{30} + \dots + {}^{98}C_{30} - {}^{46}C_{31}$$

.....

$$= {}^{99}C_{31} - {}^{46}C_{31} = {}^{99}C_p - {}^{46}C_q$$

Possible values of $(p + q)$ are 62, 83, 99, 46

$$\Rightarrow p + q = 83$$

7. Let

$$\int \frac{2 - \tan x}{3 + \tan x} dx = \frac{1}{2} (\alpha x + \log_e |\beta \sin x + \gamma \cos x|) + C$$

, where C is the constant of integration.

Then $\alpha + \frac{\gamma}{\beta}$ is equal to :

(1) 3 (2) 1

(3) 4 (4) 7

Ans. (3)

Sol. $\int \frac{2 - \tan x}{3 + \tan x} dx = \int \frac{2 \cos x - \sin x}{3 \cos x + \sin x} dx$

$$2 \cos x - \sin x = A(3 \cos x + \sin x) + B(\cos x - 3 \sin x)$$

$$3A + B = 2$$

$$A - 3B = -1$$



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$$\Rightarrow A = \frac{1}{2}, B = \frac{1}{2}$$

$$\therefore \int \frac{2 \cos x - \sin x}{3 \cos x + \sin x} dx$$

$$= \frac{x}{2} + \frac{1}{2} \ln |3 \cos x + \sin x| + C$$

$$= \frac{1}{2} (x + \ln |3 \cos x + \sin x|) + C$$

$$= \frac{1}{2} (\alpha x + \ln |\beta \sin x + \gamma \cos x|) + C$$

$$\alpha = 1, \beta = 1, \gamma = 3$$

$$\therefore \alpha + \frac{\gamma}{\beta} = 1 + \frac{3}{1} = 4$$

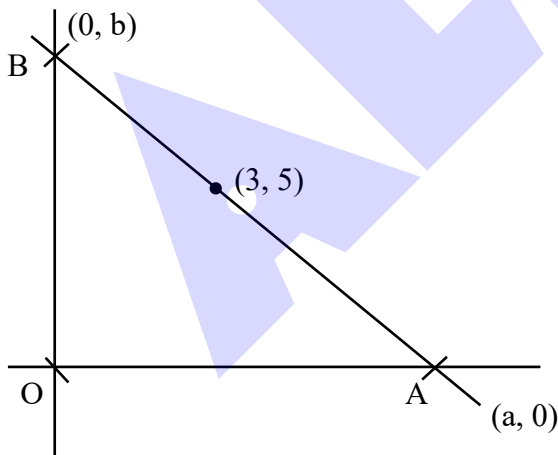
8. A variable line L passes through the point (3, 5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is :

- (1) 30 (2) 25
(3) 40 (4) 35

Ans. (1)

Sol. $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{3}{a} + \frac{5}{b} = 1 \Rightarrow b = \frac{5a}{a-3}, a > 3$$



$$A = \frac{1}{2} ab = \frac{1}{2} a \frac{5a}{a-3} = \frac{5}{2} \cdot \frac{a^2}{a-3}$$

$$= \frac{5}{2} \left(\frac{a^2 - 9 + 9}{a-3} \right)$$

$$= \frac{5}{2} \left(a + 3 + \frac{9}{a-3} \right)$$

$$= \frac{5}{2} \left(a - 3 + \frac{9}{a-3} + 6 \right) \geq 30$$

9. Let

$$|\cos \theta \cos(60 - \theta) \cos(60 + \theta)| \leq \frac{1}{8}, \theta \in [0, 2\pi]$$

Then, the sum of all $\theta \in [0, 2\pi]$, where $\cos 3\theta$ attains its maximum value, is :

- (1) 9π (2) 18π
(3) 6π (4) 15π

Ans. (3)

Sol. We know that

$$(\cos \theta) (\cos(60^\circ - \theta)) (\cos(60^\circ + \theta)) = \frac{1}{4} \cos 3\theta$$

So equation reduces to $\left| \frac{1}{4} \cos 3\theta \right| \leq \frac{1}{8}$

$$\Rightarrow |\cos 3\theta| \leq \frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} \leq \cos 3\theta \leq \frac{1}{2}$$

$$\Rightarrow \text{maximum value of } \cos 3\theta = \frac{1}{2}, \text{ here}$$

$$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}$$

$$\theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$$

As $\theta \in [0, 2\pi]$ possible values are

$$\theta = \left\{ \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9} \right\}$$

Whose sum is

$$\frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} + \frac{11\pi}{9} + \frac{13\pi}{9} + \frac{17\pi}{9} = \frac{54\pi}{9} = 6\pi$$



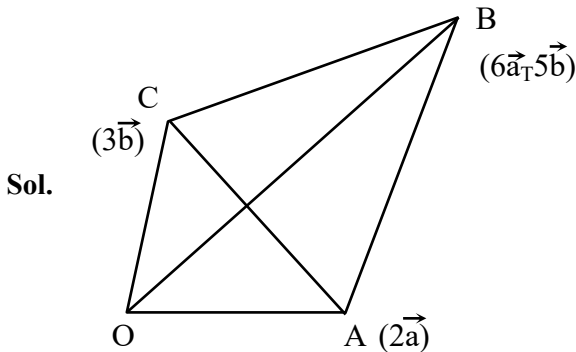
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10. Let $\vec{OA} = 2\vec{a}, \vec{OB} = 6\vec{a} + 5\vec{b}$ and $\vec{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides \vec{OA} and \vec{OC} is 15 sq. units, then the area (in sq. units) of the quadrilateral OABC is equal to :

- (1) 38 (2) 40
(3) 32 (4) 35

Ans. (4)



Sol.

Area of parallelogram having sides

$$\vec{OA} \text{ \& \ } \vec{OC} = |\vec{OA} \times \vec{OC}| = |2\vec{a} \times 3\vec{b}| = 15$$

$$6|\vec{a} \times \vec{b}| = 15$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \frac{5}{2} \dots\dots(1)$$

Area of quadrilateral

$$OABC = \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$$

$$= \frac{1}{2} |\vec{AC} \times \vec{OB}| = \frac{1}{2} |(3\vec{b} - 2\vec{a}) \times (6\vec{a} + 5\vec{b})|$$

$$= \frac{1}{2} |18\vec{b} \times \vec{a} - 10\vec{a} \times \vec{b}| = 14|\vec{a} \times \vec{b}|$$

$$= 14 \times \frac{5}{2} = 35$$

11. If the domain of the function

$$f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right) \text{ is } R - (\alpha, \beta)$$

then $12\alpha\beta$ is equal to :

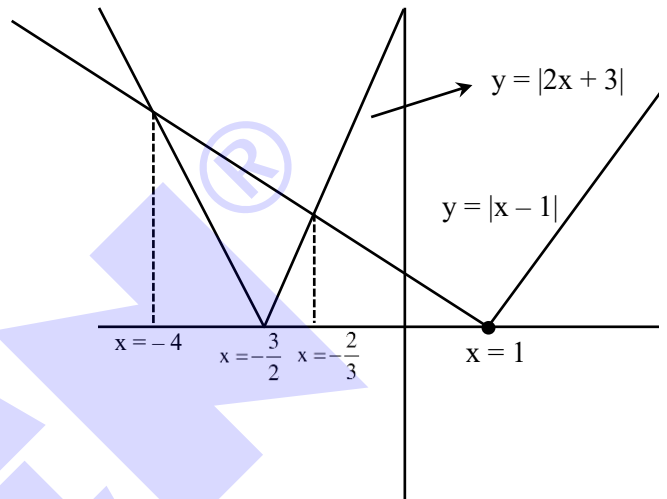
- (1) 36 (2) 24
(3) 40 (4) 32

Ans. (4)

Sol. Domain of $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is

$$2x+3 \neq 0 \text{ \& \ } x \neq \frac{-3}{2} \text{ and } \left|\frac{x-1}{2x+3}\right| \leq 1$$

$$|x-1| \leq |2x+3|$$



$$\text{For } |2x+3| \geq |x-1|$$

$$x \in (-\infty, -4] \cup \left(-\frac{2}{3}, \infty\right)$$

$$\alpha = -4 \text{ \& \ } \beta = -\frac{2}{3} : 12\alpha\beta = 32$$

12. If the sum of series

$$\frac{1}{1 \cdot (1+d)} + \frac{1}{(1+d)(1+2d)} + \dots + \frac{1}{(1+9d)(1+10d)}$$

is equal to 5, then $50d$ is equal to :

- (1) 20 (2) 5
(3) 15 (4) 10

Ans. (2)

Sol. $\frac{1}{1 \cdot (1+d)} + \frac{1}{(1+d)(1+2d)} + \dots$

$$\frac{1}{(1+9d)(1+10d)} = 5$$

$$\frac{1}{d} \left[\frac{(1+d)-1}{1 \cdot (1+d)} + \frac{(1+2d)-(1-d)}{(1+d)(1+2d)} \right] + \dots$$

$$\frac{(1+10d)-(1+9d)}{(1+9d)(1+10d)} = 5$$

$$\frac{1}{d} \left[\left(1 - \frac{1}{1+d}\right) + \left(\frac{1}{1+d} - \frac{1}{1+2d}\right) + \dots \right]$$

$$\left[\frac{1}{1+9d} - \frac{1}{1+10d} \right] = 5$$

$$\frac{1}{d} \left[1 - \frac{1}{(1+10d)} \right] = 5$$

$$\frac{10d}{1+10d} = 5d$$

$$50d = 5$$

13. Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that

$$f(1) = 40, f'(1) = 2 \text{ and } f''(1) = 4.$$

Then $a^2 + b^2 + c^2$ is equal to :

- (1) 62 (2) 73
(3) 54 (4) 51

Ans. (4)

Sol. $f(x) = ax^3 + bx^2 + cx + 41$

$$f'(x) = 3ax^2 + 2bx + cx$$

$$\Rightarrow f'(1) = 3a + 2b + c = 2 \dots (1)$$

$$f''(x) = 6ax + 2b$$

$$\Rightarrow f''(1) = 6a + 2b = 4$$

$$3a + b = 2 \dots (2)$$

$$(1) - (2)$$

$$b + c = 0 \dots (3)$$

$$f(1) = 40$$

$$a + b + c + 41 = 40$$

use (3)

$$a + 41 = 40$$

by (2)

$$-3 + b = 2 \Rightarrow b = 5 \text{ \& } c = -5$$

$$a^2 + b^2 + c^2 = 1 + 25 + 25 = 51$$

14. Let a circle passing through $(2, 0)$ have its centre at the point (h, k) . Let (x_c, y_c) be the point of intersection of the lines $3x + 5y = 1$ and $(2+c)x + 5c^2y = 1$. If $h = \lim_{c \rightarrow 1} x_c$ and $k = \lim_{c \rightarrow 1} y_c$, then the equation of the circle is :

- (1) $25x^2 + 25y^2 - 20x + 2y - 60 = 0$
(2) $5x^2 + 5y^2 - 4x - 2y - 12 = 0$
(3) $25x^2 + 25y^2 - 2x + 2y - 60 = 0$
(4) $5x^2 + 5y^2 - 4x + 2y - 12 = 0$

Ans. (1)

Sol. $(2+c)x + 5c^2 \left(\frac{1-3x}{5} \right) = 1$

$$x = \frac{1-c^2}{2+c-3c^2}, y = \frac{1-3x}{5} = \frac{c-1}{5(2+c-3c^2)}$$

$$h = \lim_{c \rightarrow 1} \frac{(1-c)(1+c)}{(1-c)(2+3c)} = \frac{2}{5}$$

$$k = \lim_{c \rightarrow 1} \frac{c-1}{-5(c-1)(3c+2)} = -\frac{1}{25}$$

Centre $\left(\frac{2}{25}, -\frac{1}{25} \right)$,

$$r = \sqrt{\left(2 - \frac{2}{25}\right)^2 + \left(0 - \frac{1}{25}\right)^2} = \sqrt{\frac{64}{25} + \frac{1}{625}}$$

$$r = \frac{\sqrt{161}}{25}$$

$$\left(x - \frac{2}{25}\right)^2 + \left(y + \frac{1}{25}\right)^2 = \frac{161}{625}$$

$$\Rightarrow 25x^2 + 25y^2 - 20x + 2y - 60 = 0$$

15. The shortest distance between the line

$$\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5} \text{ and } \frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$$

is :

- (1) $\frac{187}{\sqrt{563}}$ (2) $\frac{178}{\sqrt{563}}$



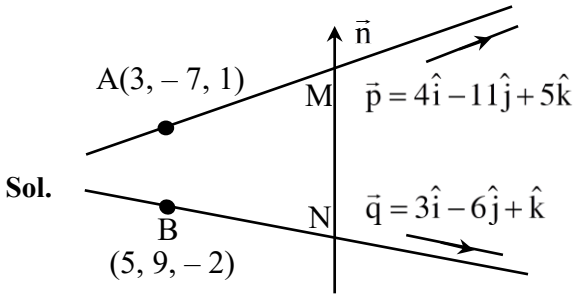
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(3) $\frac{185}{\sqrt{563}}$

(4) $\frac{179}{\sqrt{563}}$

Ans. (1)



Sol.

$\vec{n} = \vec{p} \times \vec{q}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -11 & 5 \\ 3 & -6 & 1 \end{vmatrix} = 19\hat{i} + 11\hat{j} + 9\hat{k}$$

S.d. = projection of \vec{AB} on \vec{n}

$$= \frac{|\vec{AB} \cdot \vec{n}|}{|\vec{n}|} = \frac{|(2\hat{i} + 16\hat{j} - 3\hat{k}) \cdot (19\hat{i} + 11\hat{j} + 9\hat{k})|}{\sqrt{361 + 121 + 81}}$$

$$= \frac{38 + 176 - 27}{\sqrt{563}}$$

S.d. = $\frac{187}{\sqrt{563}}$

16. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No. of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then $4x + 5y$ is equal to :

- (1) 43
- (2) 44
- (3) 47
- (4) 46

Ans. (2)

Sol. $x + y = 10$ (1)

Median = 18 = M

$$M.D. = \frac{\sum f_i |x_i - M|}{\sum f_i}$$

$$1.25 = \frac{36 + x + 2y}{40}$$

$x + 2y = 14$ (2)

by (1) & (2)

$x = 6, y = 4$

$\Rightarrow 4x + 5y = 24 + 20 = 44$

Age(x _i)	f	x _i - M	f _i x _i - M
15	5	3	15
16	8	2	16
17	5	1	5
18	12	0	0
19	x	1	x
20	y	2	2y

17. The solution of the differential equation

$(x^2 + y^2)dx - 5xy dy = 0, y(1) = 0$, is :

(1) $|x^2 - 4y^2|^5 = x^2$ (2) $|x^2 - 2y^2|^6 = x$

(3) $|x^2 - 4y^2|^6 = x$ (4) $|x^2 - 2y^2|^5 = x^2$

Ans. (1)

Sol. $(x^2 + y^2) dx = 5xy dy$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{5xy}$$

Put $y = Vx$

$$\Rightarrow V + x \frac{dV}{dx} = \frac{1 + V^2}{5V}$$

$$\Rightarrow \frac{xdV}{dx} = \frac{1 - 4V^2}{5V}$$

$$\Rightarrow \int \frac{V}{1 - 4V^2} dV = \int \frac{dx}{5x}$$

Let $1 - 4V^2 = t$

$\Rightarrow -8V dV = dt$



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$$\Rightarrow \int \frac{dt}{(-8)(t)} = \int \frac{dx}{5x}$$

$$\Rightarrow \frac{-1}{8} \ln|t| = \frac{1}{5} \ln|x| + \ln C$$

$$\Rightarrow -5 \ln|t| = 8 \ln|x| + \ln K$$

$$\Rightarrow \ln x^8 + \ln|t^5| + \ln K = 0$$

$$\Rightarrow x^8 |t^5| = C$$

$$\Rightarrow x^8 |1 - 4V^2|^5 = C$$

$$\Rightarrow x^8 \left| \frac{x^2 - 4y^2}{x^2} \right|^5 = C$$

$$\Rightarrow |x^2 - 4y^2|^5 = Cx^2$$

given $y(1) = 0$

$$\Rightarrow |1|^5 = C \Rightarrow C = 1$$

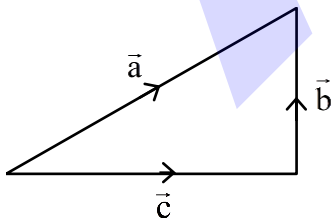
$$\Rightarrow |x^2 - 4y^2|^5 = x^2$$

18. Let three vectors $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}$,
 $\vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ form a triangle
 such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is
 $5\sqrt{6}$. if α is a positive real number, then $|\vec{c}|^2$ is :

- (1) 16 (2) 14
 (3) 12 (4) 10

Ans. (2)

Sol. $\vec{c} = \vec{a} - \vec{b}$
 $\Rightarrow (x, y, z) = (\alpha - 5, 1, -2)$
 $\Rightarrow x = \alpha - 5, y = 1, z = -2$ (1)



Area of $\Delta = 5\sqrt{6}$ (given)

$$\frac{1}{2} |\vec{a} \times \vec{c}| = 5\sqrt{6}$$

$$\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 4 & 2 \\ x & 1 & -2 \end{matrix} \right\| = 10\sqrt{6}$$

$$\Rightarrow |-10\hat{i} - \hat{j}(-2\alpha - 2x) + \hat{k}(\alpha - 4x)| = 10\sqrt{6}$$

$$\Rightarrow (2\alpha + 2\alpha - 10)^2 + (\alpha - 4\alpha + 20)^2 = 500$$

$$\Rightarrow (4\alpha - 10)^2 + (20 - 3\alpha)^2 = 500$$

$$\Rightarrow 25\alpha^2 - 80\alpha - 120\alpha = 0$$

$$\Rightarrow \alpha(25\alpha - 200) = 0$$

$$\Rightarrow \alpha = 8 \text{ (given } \alpha \text{ is +ve number)}$$

$$\Rightarrow x = \alpha - 5 = 3$$

$$|\vec{c}|^2 = x^2 + y^2 + z^2$$

$$= 9 + 1 + 4$$

$$= 14$$

19. Let α, β be the roots of the equation

$$x^2 + 2\sqrt{2}x - 1 = 0. \text{ The quadratic equation,}$$

whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is :

(1) $x^2 - 190x + 9466 = 0$

(2) $x^2 - 195x + 9466 = 0$

(3) $x^2 - 195x + 9506 = 0$

(4) $x^2 - 180x + 9506 = 0$

Ans. (3)

Sol. $x^2 + 2\sqrt{2}x - 1 = 0$

$$\alpha + \beta = -2\sqrt{2}$$

$$\alpha\beta = -1$$

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

$$= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2$$

$$= (8 + 2)^2 - 2(-1)^2$$

$$= 100 - 2 = 98$$

$$\alpha^6 + \beta^6 = (\alpha^3 + \beta^3)^2 - 2\alpha^3\beta^3$$

$$= ((\alpha + \beta)((\alpha + \beta)^2 - 3\alpha\beta))^2 - 2(\alpha\beta)^3$$



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$$= (-2\sqrt{2}(8+3))^2 + 2$$

$$= (8)(121) + 2 = 970$$

$$\frac{1}{10}(\alpha^6 + \beta^6) = 97$$

$$x^2 - (98 + 97)x + (98)(97) = 0$$

$$\Rightarrow x^2 - 195x + 9506 = 0$$

20. Let $f(x) = x^2 + 9$, $g(x) = \frac{x}{x-9}$ and

$a = fog(10)$, $b = gof(3)$. If e and l denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$, then $8e^2 + l^2$ is equal to.

- (1) 16
- (2) 8
- (3) 6
- (4) 12

Ans. (2)

Sol. $f(x) = x^2 + 9$ $g(x) = \frac{x}{x-9}$

$$a = f(g(10)) = f\left(\frac{10}{10-9}\right)$$

$$= f(10) = 109$$

$$b = g(f(3)) = g(9+9)$$

$$= g(18) = \frac{18}{9} = 2$$

$$E: \frac{x^2}{109} + \frac{y^2}{2} = 1$$

$$e^2 = 1 - \frac{2}{109} = \frac{107}{109}$$

$$l = \frac{2(2)}{\sqrt{109}} = \frac{4}{\sqrt{109}}$$

$$8e^2 + l^2 = \frac{8(107)}{109} + \frac{16}{109}$$

$$= 8$$

SECTION-B

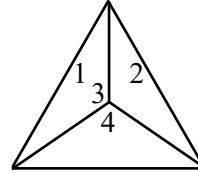
21. Let a , b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose

four faces are marked 1, 2, 3, 4. If the probability that $ax^2 + bx + c = 0$ has all real roots is $\frac{m}{n}$,

$gcd(m, n) = 1$, then $m + n$ is equal to _____.

Ans. (19)

Sol. $a, b, c \in \{1, 2, 3, 4\}$



Tetrahedral dice $ax^2 + bx + c = 0$

has all real roots

$$\Rightarrow D \geq 0$$

$$\Rightarrow b^2 - 4ac \geq 0$$

$$\text{Let } b = 1 \Rightarrow 1 - 4ac \geq 0 \text{ (Not feasible)}$$

$$b = 2 \Rightarrow 4 - 4ac \geq 0$$

$$1 \geq ac \Rightarrow a = 1, c = 1,$$

$$b = 3 \Rightarrow 9 - 4ac \geq 0$$

$$\frac{9}{4} \geq ac$$

$$\Rightarrow a = 1, c = 1$$

$$\Rightarrow a = 1, c = 2$$

$$\Rightarrow a = 2, c = 1$$

$$b = 4 \Rightarrow 16 - 4ac \geq 0$$

$$4 \geq ac$$

$$\Rightarrow a = 1, c = 1$$

$$\Rightarrow a = 1, c = 2 \Rightarrow a = 2, c = 1$$

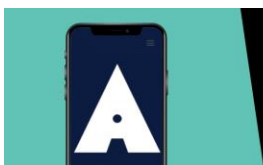
$$\Rightarrow a = 1, c = 3 \Rightarrow a = 3, c = 1$$

$$\Rightarrow a = 1, c = 4 \Rightarrow a = 4, c = 1$$

$$\Rightarrow a = 2, c = 2$$

$$\text{Probability} = \frac{12}{(4)(4)(4)} = \frac{3}{16} = \frac{m}{n}$$

$$m + n = 19$$



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22. The sum of the square of the modulus of the elements in the set

$$\{z = a + ib : a, b \in \mathbb{Z}, z \in \mathbb{C}, |z-1| \leq 1, |z-5| \leq |z-5i|\}$$

is _____.

Ans. (9)

Sol. $|z-1| \leq 1$

$$\Rightarrow |(x-1) + iy| \leq 1$$

$$\Rightarrow \sqrt{(x-1)^2 + y^2} \leq 1$$

$$\Rightarrow (x-1)^2 + y^2 \leq 1 \dots\dots\dots (1)$$

Also $|z-5| \leq |z-5i|$

$$(x-5)^2 + y^2 \leq x^2 + (y-5)^2$$

$$-10x \leq -10y$$

$$\Rightarrow x \geq y \dots\dots\dots (2)$$

Solving (1) and (2)

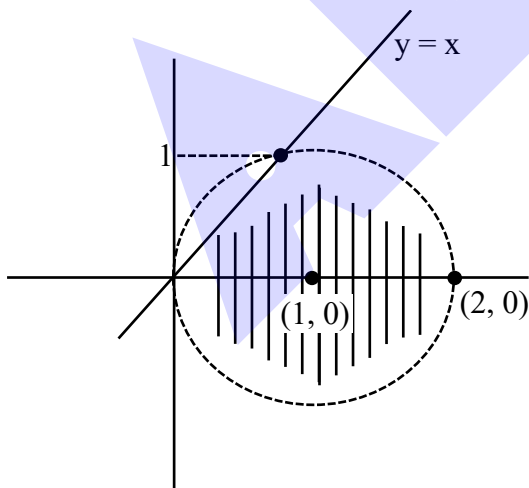
$$\Rightarrow (x-1)^2 + x^2 = 1$$

$$\Rightarrow 2x^2 - 2x = 0$$

$$\Rightarrow x(x-1) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1$$

$$y = 0 \text{ or } y = 1$$



Given $x, y \in \mathbb{I}$

Points (0, 0), (1, 0), (2, 0), (1, 1), (1, -1)

to find

$$|z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 + |z_5|^2$$

$$= 0 + 1 + 4 + 1 + 1 + 1 + 1 = 9$$

23. Let the set of all positive values of λ , for which the point of local minimum of the function

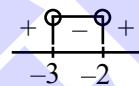
$$(1 + x(\lambda^2 - x^2)) \text{ satisfies } \frac{x^2 + x + 2}{x^2 + 5x + 6} < 0, \text{ be } (\alpha, \beta).$$

Then $\alpha^2 + \beta^2$ is equal to _____.

Ans. (39)

Sol. $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$

$$\Rightarrow \frac{1}{(x+2)(x+3)} < 0$$



$$x \in (-3, -2) \dots\dots\dots (1)$$

$$f(x) = 1 + x(\lambda^2 - x^2)$$

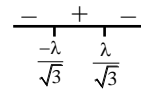
Finding local minima

$$f'(x) = (\lambda^2 - x^2) + (-2x).x$$

Put $f'(x) = 0$

$$\Rightarrow \lambda^2 = 3x^2$$

$$\Rightarrow x = \pm \frac{\lambda}{\sqrt{3}}$$



Local min Local max

We want local min

$$\Rightarrow x = \frac{-\lambda}{\sqrt{3}}$$

from (1)

$$x \in (-3, -2)$$

$$-3 < \frac{-\lambda}{\sqrt{3}} < -2$$

$$3\sqrt{3} > \lambda > 2\sqrt{3}$$

$$\alpha = 2\sqrt{3}, \beta = 3\sqrt{3}$$

$$\alpha^2 + \beta^2 = 12 + 27 = 39$$

24. Let

$$\lim_{n \rightarrow \infty} \left(\frac{n}{\sqrt{n^4+1}} - \frac{2n}{(n^2+1)\sqrt{n^4+1}} + \frac{n}{\sqrt{n^4+16}} - \frac{8n}{(n^2+4)\sqrt{n^4+16}} + \dots + \frac{n}{\sqrt{n^4+n^4}} - \frac{2n \cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}} \right) \text{ be } \frac{\pi}{k},$$

using only the principal values of the inverse trigonometric functions. Then k^2 is equal to _____.

Ans. (32)

Sol.
$$\sum_{r=1}^{\infty} \frac{n}{\sqrt{n^4+r^4}} - \frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}$$

$$\sum_{r=1}^{\infty} \frac{\frac{1}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^4}} - \frac{2\left(\frac{1}{n}\right)\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}$$

$$\Rightarrow \int_0^1 \frac{dx}{\sqrt{1+x^4}} - \frac{2x^2 dx}{(1+x^2)\sqrt{1+x^4}}$$

$$\Rightarrow \int_0^1 \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}} dx$$

$$\Rightarrow \int_0^1 \frac{\frac{1}{x^2}-1}{\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}} dx$$

$$\Rightarrow -\int_0^1 \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}} dx$$

$$x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2} dx = dt$$

$$\Rightarrow -\int_{\infty}^2 \frac{dt}{t\sqrt{t^2-2}}$$

$$\Rightarrow -\int_{\infty}^2 \frac{t dt}{t^2\sqrt{t^2-2}}$$

$$\text{take } t^2 - 2 = \alpha^2$$

$$t dt = \alpha d\alpha$$

$$\Rightarrow -\int_{\infty}^{\sqrt{2}} \frac{\alpha d\alpha}{(\alpha^2+2)\alpha}$$

$$\Rightarrow -\int_{\infty}^{\sqrt{2}} \frac{d\alpha}{\alpha^2+2}$$

$$\Rightarrow \left. \frac{-1}{\sqrt{2}} \tan^{-1} \frac{\alpha}{\sqrt{2}} \right]_{\infty}^{\sqrt{2}}$$

$$\Rightarrow \frac{-1}{\sqrt{2}} \{ \tan^{-1} 1 \} + \frac{1}{\sqrt{2}} \tan^{-1} \infty$$

$$\Rightarrow \frac{1}{\sqrt{2}} \left\{ \frac{\pi}{2} - \frac{\pi}{4} \right\}$$

$$\Rightarrow \frac{\pi}{4\sqrt{2}} = \frac{\pi}{K}$$

$$\text{So } K = 4\sqrt{2}$$

$$K^2 = 32$$

25. The remainder when 428^{2024} is divided by 21 is _____.

Ans. (1)

Sol. $(428)^{2024} = (420 + 8)^{2024}$

$$= (21 \times 20 + 8)^{2024}$$

$$= 21m + 8^{2024}$$

$$\text{Now } 8^{2024} = (8^2)^{1012}$$


$$= (64)^{1012}$$

$$= (63 + 1)^{1012}$$

$$= (21 \times 3 + 1)^{1012}$$

$$= 21n + 1$$

\Rightarrow Remainder is 1.



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26. Let $f: (0, \pi) \rightarrow \mathbb{R}$ be a function given by

$$f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b}{a} |\tan x|}, & \frac{\pi}{2} < x < \pi \end{cases}$$

Where $a, b \in \mathbb{Z}$. If f is continuous at $x = \frac{\pi}{2}$, then

$a^2 + b^2$ is equal to _____.

Ans. (81)

Sol. LHL at $x = \frac{\pi}{2}$

$$\lim_{x \rightarrow \frac{\pi}{2}^-} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} = \left(\frac{8}{7}\right)^0 = 1$$

RHL at $x = \frac{\pi}{2}$

$$\lim_{x \rightarrow \frac{\pi}{2}^+} (1 + |\cot x|)^{\frac{b}{a} |\tan x|}$$

$$= e^{\lim_{x \rightarrow \frac{\pi}{2}^+} |\cot x| \frac{b}{a} |\tan x|} = e^{\frac{b}{a}}$$

$$\Rightarrow 1 = a - 8 = e^{\frac{b}{a}}$$

$$\Rightarrow a = 9, b = 0$$

$$\Rightarrow a^2 + b^2 = 81$$

27. Let A be a non-singular matrix of order 3. If $\det(3\text{adj}(2\text{adj}((\det A)A))) = 3^{-13} \cdot 2^{-10}$ and $\det(3\text{adj}(2A)) = 2^m \cdot 3^n$, then $|3m + 2n|$ is equal to _____.

Ans. (14)

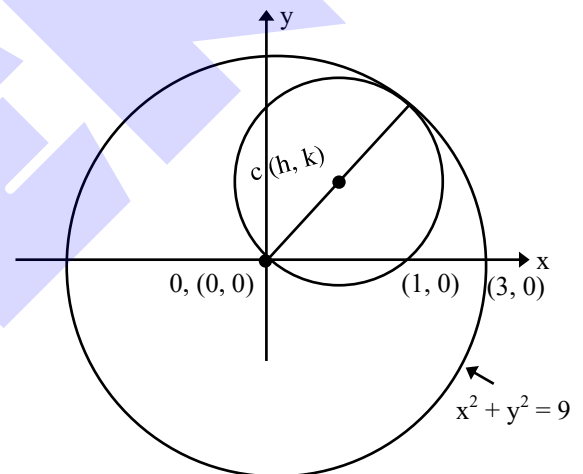
Sol. $|3 \text{adj}(2\text{adj}(|A|A))| = |3\text{adj}(2|A|^2 \text{adj}(A))|$
 $= |3 \cdot 2^2 |A|^4 \text{adj}(\text{adj}(A))| = 2^6 3^3 |A|^{12} |A|^4$
 $= 2^6 3^3 |A|^{16} = 2^{-10} 3^{-13}$
 $\Rightarrow |A|^{16} = 2^{-16} 3^{-16} \Rightarrow |A| = 2^{-1} 3^{-1}$

Now $|3\text{adj}(2A)| = |3 \cdot 2^2 \text{adj}(A)|$
 $= 2^6 3^3 |A|^2 = 2^{-m} 3^{-n}$
 $\Rightarrow 2^6 3^3 2^{-2} 3^{-2} = 2^{-m} 3^{-n}$
 $\Rightarrow 2^{-m} 3^{-n} = 2^4 3^1$
 $\Rightarrow m = -4, n = -1$
 $\Rightarrow |3m + 2n| = |-12 - 2| = 14$

28. Let the centre of a circle, passing through the point $(0, 0)$, $(1, 0)$ and touching the circle $x^2 + y^2 = 9$, be (h, k) . Then for all possible values of the coordinates of the centre (h, k) , $4(h^2 + k^2)$ is equal to _____.

Ans. (9)

Sol.



$$(x - h)^2 + (y - k)^2 = h^2 + k^2$$

$$x^2 + y^2 - 2hx - 2ky = 0$$

\therefore passes through $(1, 0)$

$$\Rightarrow 1 + 0 - 2h = 0$$

$$\Rightarrow h = 1/2$$

$$\therefore OC = \frac{OP}{2}$$

$$\sqrt{\left(\frac{1}{2}\right)^2 + k^2} = \frac{3}{2}$$

$$\frac{1}{4} + k^2 = \frac{9}{4}$$

$$k^2 = 2$$

$$k = \pm \sqrt{2}$$

∴ Possible coordinate of

$$c(h, k) \left(\frac{1}{2}, \sqrt{2} \right) \left(\frac{1}{2}, -\sqrt{2} \right)$$

$$4(h^2 + k^2) = 4 \left(\frac{1}{4} + 2 \right) = 4 \left(\frac{9}{4} \right) = 9$$

29. If a function f satisfies $f(m + n) = f(m) + f(n)$ for all $m, n \in \mathbb{N}$ and $f(1) = 1$, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$ is equal to _____.

Ans. (1010)

Sol. $f(m + n) = f(m) + f(n)$

$$\Rightarrow f(x) = kx$$

$$\Rightarrow f(1) = 1$$

$$\Rightarrow k = 1$$

$$f(x) = x$$

Now

$$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$$

$$\Rightarrow \sum_{k=1}^{2022} (\lambda + k) \leq (2022)^2$$

$$\Rightarrow 2022\lambda + \frac{2022 \times 2023}{2} \leq (2022)^2$$

$$\Rightarrow \lambda \leq 2022 - \frac{2023}{2}$$

$$\Rightarrow \lambda \leq 1010.5$$

∴ largest natural no. λ is 1010.

30. Let $A = \{2, 3, 6, 7\}$ and $B = \{4, 5, 6, 8\}$. Let R be a relation defined on $A \times B$ by $(a_1, b_1) R (a_2, b_2)$ is and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is _____.

Ans. (25)

Sol. $A = \{2, 3, 6, 7\}$

$B = \{2, 5, 6, 8\}$

$(a_1, b_1) R (a_2, b_2)$

$a_1 + a_2 = b_1 + b_2$

- | | |
|---------------------|---------------------|
| 1. (2, 4) R (6, 4) | 2. (2, 4) R (7, 5) |
| 3. (2, 5) R (7, 4) | 4. (3, 4) R (6, 5) |
| 5. (3, 5) R (6, 4) | 6. (3, 5) R (7, 5) |
| 7. (3, 6) R (7, 4) | 8. (3, 4) R (7, 6) |
| 9. (6, 5) R (7, 8) | 10. (6, 8) R (7, 5) |
| 11. (7, 8) R (7, 6) | 12. (6, 8) R (6, 4) |
| 13. (6, 6) R (6, 6) | |

× 2

Total 24 + 1 = 25



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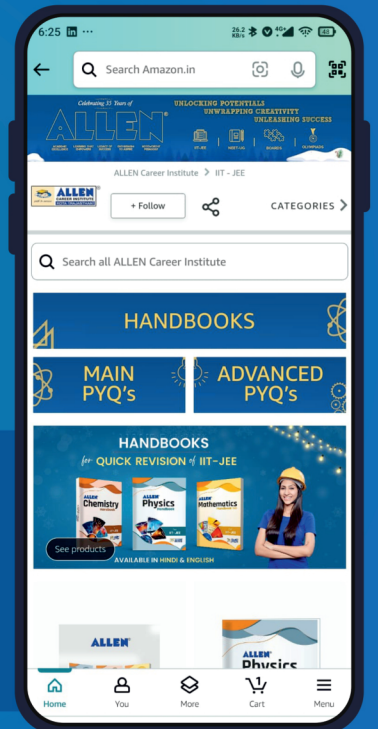
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