# FINAL JEE–MAIN EXAMINATION – APRIL, 2024

**(Held On Monday 08th April, 2024)** **TIME : 3 : 00 PM to 6 : 00 PM**

**MATHEMATICS** | **TEST PAPER WITH SOLUTION**

## SECTION-A

**1.** If the image of the point (-4, 5) in the line $x + 2y = 2$ lies on the circle $(x + 4)^2 + (y-3)^2 = r^2$, then r is equal lo :

(1) 1 (2) 2

(3) 75 (4) 3

**Ans. (2)**

**Sol.** Image of point (–4, 5)

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

Line : $x + 2y - 2 = 0$

$$\frac{x+4}{1}=\frac{y-5}{2}=-2\left(\frac{-4+10-2}{1^2+2^2}\right)$$

$$=\frac{-8}{5}$$

$$x=-4-\frac{8}{5}=-\frac{28}{5}$$

$$y=-\frac{16}{5}+5=\frac{9}{5}$$

Point lies on circle $(x + 4)^2 + (y - 3)^2 = r^2$

$$\frac{64}{25}+\left(\frac{9}{5}-3\right)^2=r^2$$

$$\frac{100}{25}=r^2,\ \boxed{r=2}$$

**2.** Let $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+3\hat{j}-5\hat{k}$ and $\vec{c}=3\hat{i}-\hat{j}+\lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b}+\vec{c}$. If $\vec{r}\cdot\vec{a}=3$, then $3\lambda$ is equal to :

(1) 27 (2) 25

(3) 25 (4) 21

**Ans. (2)**

**Sol.** $\vec{r}=k(\vec{b}+\vec{c})$

$\vec{r}\cdot\vec{a}=3$

$\vec{r}.\vec{a}=k(\vec{b}.\vec{a}+\vec{c}.\vec{a})$

$3 = k(2 + 6 - 15 + 3 - 2 + 3\lambda)$

$3 = k(-6 + 3\lambda)$ ...(1)

$\vec{r}=k(5\hat{i}+2\hat{j}-(5-\lambda)\hat{k})$

$|\vec{r}|=k\sqrt{25+4+25+\lambda^2-10\lambda}=1$ ...(2)

$k=\dfrac{3}{-6+3\lambda}=\dfrac{1}{-2+\lambda}$ put in (2)

$4+\lambda^2-4\lambda=54+\lambda^2-10\lambda$

$6\lambda = 50$

$3\lambda = 25$

**3.** If $\alpha\neq a,\ \beta\neq b,\ \gamma\neq c$ and $\begin{vmatrix}\alpha & b & c\\ a & \beta & c\\ a & b & \gamma\end{vmatrix}=0$, then

$$\frac{a}{\alpha-a}+\frac{b}{\beta-b}+\frac{\gamma}{\gamma-c}$$ is equal to :

(1) 2 (2) 3

(3) 0 (4) 1

**Ans. (3)**

**Sol.** $R_1\to R_1-R_2,\ R_2\to R_2-R_3$

$$\begin{vmatrix}\alpha-a & b-\beta & 0\\ 0 & \beta-b & c-\gamma\\ a & b & \gamma\end{vmatrix}=0$$

$(\alpha-a)(\gamma(\beta-b)-b(c-\gamma))-(b-\beta)(-a(c-\gamma))=0$

$\gamma(\alpha-a)(\beta-b)-b(\alpha-a)(c-\gamma)+a(b-\beta)(c-\gamma)$

$$\frac{\gamma}{\gamma-c}+\frac{b}{\beta-b}+\frac{a}{\alpha-a}=0$$

**4.** In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the $4^{th}$, $6^{th}$ and $8^{th}$ terms is :-

(1) 96 (2) 78

(3) 91 (4) 84

**Ans. (3)**

**Sol.** $T_2 + T_6 = \frac{70}{3}$

$ar + ar^5 = \frac{70}{3}$

$T_3 \cdot T_5 = 49$

$ar^2 \cdot ar^4 = 49$

$a^2r^6 = 49$

$ar^3 = +7,\ a = \frac{7}{r^3}$

$ar(1 + r^4) = \frac{70}{3}$

$\frac{7}{r^2}(1+r^4) = \frac{70}{3}, r^2 = t$

$\frac{1}{t}(1+t^2) = \frac{10}{3}$

$3t^2 - 10t + 3 = 0$

$t = 3, \frac{1}{3}$

Increasing G.P. $r^2 = 3,\ r = \sqrt{3}$

$T_4 + T_6 + T_8$

$= ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4)$

$= 7(1 + 3 + 9) = 91$

**5.** The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to :

(1) 175 (2) 181

(3) 177 (4) 179

**Ans. (4)**

**Sol.** AA, MM, TT, H, I, C, S, E

(1) All distinct

${}^8C_5 \rightarrow 56$

(2) 2 same, 3 different

${}^3C_1 \times {}^7C_3 \rightarrow 105$

(3) 2 same $I^{st}$ kind, 2 same $2^{nd}$ kind, 1 different

${}^3C_2 \times {}^6C_1 \rightarrow 18$

Total $\rightarrow 179$

**6.** The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely imaginary, is equal to

(1) $2\pi$ (2) $3\pi$

(3) $5\pi$ (4) $4\pi$

**Ans. (2)**

**Sol.** $Z = \frac{1 + i\cos\theta}{1 - 2i\cos\theta}$

$Z = -\bar{Z} \Rightarrow \frac{1+i\cos\theta}{1-2i\cos\theta} = -\left(\overline{\frac{1+i\cos\theta}{1-2i\cos\theta}}\right)$

$(1+i\cos\theta)(\overline{1-2i\cos\theta}) = -(1-2i\cos\theta)(\overline{1+i\cos\theta})$

$(1+i\cos\theta)(1+2i\cos\theta) = -(1-2i\cos\theta)(1-i\cos\theta)$

$1 + 3i\cos\theta - 2\cos^2\theta = -(1 - 3i\cos\theta - 2\cos^2\theta)$

$2 - 4\cos^2\theta = 0$

$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

sum $= 3\pi$

**7.** If the system of equations $x + 4y - z = \lambda$, $7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to :

(1) 2 (2) –3

(3) 3 (4) –2

**Ans. (2)**

**Sol.** $\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$

$\Rightarrow (18-\mu) - 4(14-5\mu) - (7-45) = 0 \Rightarrow \mu = 0$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ (For infinite solution)

$\Delta_x = \begin{vmatrix} \lambda & 4 & -1 \\ -3 & 9 & \mu \\ -1 & 1 & 2 \end{vmatrix} = 0$

$\lambda(18 - \mu) - 4(-6 + \mu) - 1(-3 + 9) = 0$

$18\lambda + 24 - 6 = 0 \Rightarrow \lambda = -1$

**8.** If the shortest distance between the lines $\frac{x-\lambda}{2}=\frac{y-4}{3}=\frac{z-3}{4}$ and $\frac{x-2}{4}=\frac{y-4}{6}=\frac{z-7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of $\lambda$ is :

(1) $-\frac{13}{25}$  (2) $\frac{13}{25}$

(3) 1  (4) $-1$

**Ans. (3)**

**Sol.** $\left.\begin{array}{l}\bar{r}_1=(\lambda\hat{i}+4\hat{j}+3\hat{k})+\alpha(2\hat{i}+3\hat{j}+4\hat{k})\\ \bar{r}_2=(2\hat{i}+4\hat{j}+7\hat{k})+\beta(2\hat{i}+3\hat{j}+4\hat{k})\end{array}\right\}\begin{array}{l}\bar{b}=2\hat{i}+3\hat{j}+4\hat{k}\\ \bar{a}_1+\lambda\hat{i}+4\hat{j}+3\hat{k}\\ \bar{a}_2=2\hat{i}+4\hat{j}+7\hat{k}\end{array}$

Shortest dist. $=\frac{|\bar{b}\times(\bar{a}_2-\bar{a}_1)|}{|b|}=\frac{13}{\sqrt{29}}$

$$\frac{|(2\hat{i}+3\hat{j}+4\hat{k})\times((2-\lambda)\hat{i}+4\hat{k})|}{\sqrt{29}}=\frac{13}{\sqrt{29}}$$

$|-8\hat{j}-3(2-\lambda)\hat{k}+12\hat{i}+4(2-\lambda)\hat{j}|=13$

$|12\hat{i}-4\lambda\hat{j}+(3\lambda-6)\hat{k}|=13$

$144+16\lambda^2+(3\lambda-6)^2=169$

$16\lambda^2+(3\lambda-6)^2=25=\lambda\Rightarrow=1$

**9.** If the value of $\frac{3\cos 36^\circ+5\sin 18^\circ}{5\cos 36^\circ-3\sin 18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where a, b, c are natural numbers and gcd(a, c) = 1, then a + b + c is equal to :

(1) 50  (2) 40

(3) 52  (4) 54

**Ans. (3)**

**Sol.** $$\frac{3\left(\frac{\sqrt{5}+1}{4}\right)+5\left(\frac{\sqrt{5}-1}{4}\right)}{5\left(\frac{\sqrt{5}+1}{4}\right)-3\left(\frac{\sqrt{5}-1}{4}\right)}=\frac{8\sqrt{5}-2}{2\sqrt{5}+8}$$

$$=\frac{4\sqrt{5}-1}{\sqrt{5}+4}\times\frac{\sqrt{5}-4}{\sqrt{5}-4}$$

$$=\frac{20-16\sqrt{5}-\sqrt{5}+4}{-11}$$

$=\frac{17\sqrt{5}-24}{11}\Rightarrow a=17, b=27, c=11$

$a+b+c=52$

**10.** Let y = y(x) be the solution curve of the differential equation $\sec y\frac{dy}{dx}+2x\sin y=x^3\cos y$, $y(1)=0$. Then $y(\sqrt{3})$ is equal to :

(1) $\frac{\pi}{3}$  (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$  (4) $\frac{\pi}{12}$

**Ans. (3)**

**Sol.** $\sec^2 y\frac{dy}{dx}+2x\sin y\sec y=x^3\cos y\sec y$

$\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$

$\tan y=t\Rightarrow\sec^2 y\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dt}{dx}+2xt=x^3$, If $=e^{\int 2x\,dx}=e^{x^2}$

$te^{x^2}=\int x^3.e^{x^2}dx+c$

$x^2=Z\Rightarrow t.e^Z=\frac{1}{2}\int e^Z.ZdZ=\frac{1}{2}[e^Z.Z-e^Z]+c$

$2\tan y=(x^2-1)+2ce^{-x^2}$

$y(1)=0\Rightarrow c=0\Rightarrow y(\sqrt{3})=\frac{\pi}{4}$

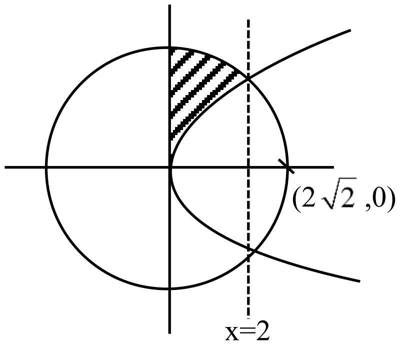
**11.** The area of the region in the first quadrant inside the circle $x^2+y^2=8$ and outside the pnrabola $y^2=2x$ is equal to :

(1) $\frac{\pi}{2}-\frac{1}{3}$  (2) $\pi-\frac{2}{3}$

(3) $\frac{\pi}{2}-\frac{2}{3}$  (4) $\pi-\frac{1}{3}$

**Ans. (2)**

**Sol.**




Required area = Ar(circle from 0 to 2) – ar(para from 0 to 2)

$$= \int_0^2 \sqrt{8-x^2}\,dx - \int_0^2 \sqrt{2x}\,dx$$

$$= \left[\frac{x}{2}\sqrt{8-x^2} + \frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}}\right]_0^2 - \sqrt{2}\left[\frac{x\sqrt{x}}{3/2}\right]_0^2$$

$$= \frac{2}{2}\sqrt{8-4} + \frac{8}{2}\sin^{-1}\frac{2}{2\sqrt{2}} - \frac{2\sqrt{2}}{3}\left(2\sqrt{2}-0\right)$$

$$\Rightarrow\ 2 + 4\cdot\frac{\pi}{4} - \frac{8}{3} = \pi - \frac{2}{3}$$

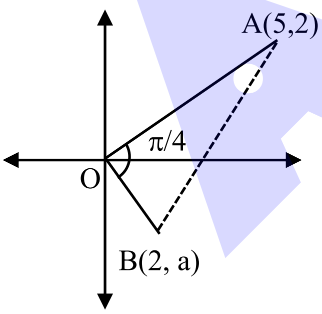
**12.** If the line segment joining the points (5, 2) and (2, a) subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is :

(1) 6 (2) 8

(3) 2 (4) 4

**Ans. (4)**

**Sol.**




$$m_{OA} = \frac{2}{5}$$

$$m_{OB} = \frac{a}{2}$$

$$\tan\frac{\pi}{4} = \left|\frac{\frac{2}{5}-\frac{a}{2}}{\ }\right|$$

$$1 = \left|\frac{4-5a}{10+2a}\right|$$

$$4 - 5a = \pm(10+2a)$$

$4 - 5a = 10 + 2a$ | $4 - 5a = -10 - 2a$

$\Rightarrow 7a + 6 = 0$ | $3a = 14$

$\Rightarrow a = -\frac{6}{7}$ | $a = +\frac{14}{3}$

$$-\frac{6}{7}\times\frac{14}{3} = -4$$

**13.** Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that

$$\left(\vec{a}+\vec{b}\right)\times\vec{c} = \vec{c}\times\left(-2\vec{a}+3\vec{b}\right).$$

If $\left(2\vec{a}+3\vec{b}\right)\cdot\vec{c} = 1670$, then $|\vec{c}|^2$ is equal to :

(1) 1627 (2) 1618

(3) 1600 (4) 1609

**Ans. (2)**

**Sol.** $\left(\vec{a}+\vec{b}\right)\times\vec{c} - \vec{c}\times\left(-2\vec{a}+3\vec{b}\right) = 0$

$$\left(\vec{a}+\vec{b}\right)\times\vec{c} + \left(-2\vec{a}+3\vec{b}\right)\times\vec{c} = 0$$

$$\Rightarrow \left(\vec{a}+\vec{b}\right) - 2\vec{a} + 3\vec{b})\times\vec{c} = 0$$

$$\Rightarrow \vec{c} = \lambda(4\vec{b} - \vec{a})$$

$$\Rightarrow\ = \lambda\left(44\hat{i} - 4\hat{j} + 4\hat{k} - 4\hat{i} + \hat{j} - \hat{k}\right)$$

$$= \lambda\left(40\hat{i} - 3\hat{j} + 3\hat{k}\right)$$

Now

$$\left(8\hat{i} - 2\hat{j} + 2\hat{k} + 33\hat{i} - 3\hat{j} + 3\hat{k}\right).\lambda(40\hat{i} - 3\hat{j} + 3\hat{k}) = 1670$$

$$\Rightarrow \left(41\hat{i} - 5\hat{j} + 5\hat{k}\right).(40\hat{i} - 3\hat{j} + 3\hat{k})\times\lambda = 1670\ )$$

$$\Rightarrow (1640 + 15 + 15)\lambda = 1670 \Rightarrow \lambda = 1$$

so $\vec{c} = 40\hat{i} - 3\hat{j} - 3\hat{k}$

$$\Rightarrow |\vec{c}|^2 = 1600 + 9 + 9 = 1618$$

**14.** If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $a > 0$ has a local maximum at $x = \alpha$ and a local minimum $x = \alpha^2$, then $\alpha$ and $\alpha^2$ are the roots of the equation :

(1) $x^2 - 6x + 8 = 0$ (2) $8x^2 + 6x - 8 = 0$

(3) $8x^2 - 6x + 1 = 0$ (4) $x^2 + 6x + 8 = 0$

**Ans. (1)**

**Sol.** $f'(x) = 6x^2 - 18ax + 12a^2 = 0 \begin{cases} \alpha \\ \alpha^2 \end{cases}$

$\alpha + \alpha^2 = 3a$ & $\alpha \times \alpha^2 = 2a^2$

$\downarrow$

$(\alpha + \alpha^2)^3 = 27a^3$

$\Rightarrow 2a^2 + 4a^4 + 3(3a)(2a^2) = 27a^3$

$\Rightarrow 2 + 4a^2 + 18a = 27a$

$\Rightarrow 4a^2 - 9a + 2 = 0$

$\Rightarrow 4a^2 - 8a - a + 2 = 0$

$\Rightarrow (4a - 1)(a - 2) = 0 \Rightarrow a = 2$

so $6x^2 - 36x + 48 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$ (1)

If we take $a = \frac{1}{4}$ then $\alpha = \frac{1}{2}$ which is not possible

**15.** There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{5}{12}$

**Ans. (1)**

**Sol.**

| X | Y | Z |
|---|---|---|
| 5 one & 4 five | 4 one & 5 five | 3 one & 6 five |

$$P = \frac{4/9}{5/9 + 4/9 + 3/9} = \frac{4}{12} = \frac{1}{3}$$

**16.** Let $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. Then $e^{\alpha}$ and $e^{-\alpha}$ are the roots of the equation :

(1) $2x^2 - 5x + 2 = 0$ (2) $x^2 - 2x - 8 = 0$

(3) $2x^2 - 5x - 2 = 0$ (4) $x^2 + 2x - 8 = 0$

**Ans. (1)**

**Sol.** $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$

Let $e^x - 1 = t^2$

$e^x\,dx = 2t\,dt$

$= \int \frac{2dt}{t^2 + 1}$

$= 2\tan^{-1} t$

$= 2\tan^{-1}\left(\sqrt{e^x - 1}\right)\Big|_{\alpha}^{\log_e^4}$

$= 2\left[\tan^{-1}\sqrt{3} - \tan^{-1}\sqrt{e^{\alpha} - 1}\right] = \frac{\pi}{6}$

$= \frac{\pi}{3} - \tan^{-1}\sqrt{e^{\alpha} - 1} = \frac{\pi}{12}$

$\Rightarrow \tan^{-1}\sqrt{e^{\alpha} - 1} = \frac{\pi}{4}$

$e^{\alpha} = 2 \quad e^{-\alpha} = \frac{1}{2}$

$x^2 - \left(2 + \frac{1}{2}\right)x + 1 = 0$

$2x^2 - 5x + 2 = 0$

**17.** Let $f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x + a & \text{if } 0 < x \le a \end{cases}$

where $a > 0$ and $g(x) = (f|x|) - |f(x)|)/2$.

Then the function $g : [-a, a] \to [-a, a]$ is

(1) neither one-one nor onto.

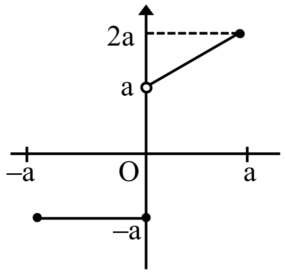
(2) both one-one and onto.

(3) one-one.

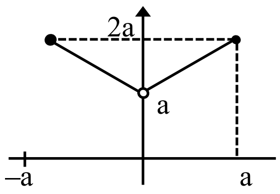
(4) onto

**Ans. (1)**

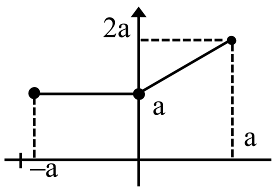
**Sol.** $y = f(x)$



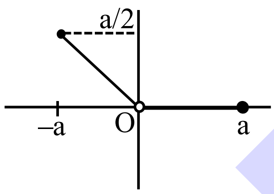
$y = f|x|$



$y = |f(x)|$



$g(x) = \dfrac{f(|x|) - |f(x)|}{2}$



**18.** Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$ Let R be a relation on $A \times B$ define by $(a, b)R(c, d)$ if and only if $3ad - 7bc$ is an even integer. Then the relation R is

(1) reflexive but not symmetric.

(2) transitive but not symmetric.

(3) reflexive and symmetric but not transitive.

(4) an equivalence relation.

**Ans. (3)**

**Sol.** $A = \{2, 3, 6, 8, 9, 11\}$ $\quad (a, b)R(c, d)$

$B = \{1, 4, 5, 10, 15\}$ $\quad 3ad - 7bc$

Reflexive : $(a, b)\ R(a, b)$

$\Rightarrow 3ab - 7ba = -4ab$ always even so it is reflexive.

Symmetric : If $3ad - 7bc$ = Even

Case-I : odd odd

Case-II : even even

$(c, d)\ R(a, b) \Rightarrow 3bc - 3ab$

Case-I : odd odd

Case-II : even even

so symmetric relation

Transitive :

Set $(3, 4)R(6, 4)$ Satisfy relation

Set $(6, 4)R(3, 1)$ Satisfy relation

but $(3, 4)\ R(3, 1)$ does not satisfy relation

so not transitive.

**19.** For $a, b > 0$, let

$$f(x) = \begin{cases} \dfrac{\tan((a+1)x) + b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \dfrac{\sqrt{ax + b^2x^2} - \sqrt{ax}}{b\sqrt{a}\,x\sqrt{x}}, & x > 0 \end{cases}$$

be a continous function at $x = 0$. Then $\dfrac{b}{a}$ is equal to

(1) 5 (2) 4

(3) 8 (4) 6

**Ans. (4)**

**Sol.** $\lim\limits_{x\to 0} f(x) = f(0) = 3$

$$\lim_{x\to 0^+} \frac{\sqrt{ax+b^2x^2} - \sqrt{ax}}{b\sqrt{a}\ x\sqrt{x}} = 3$$

$$\lim_{x\to 0^+} \frac{ax + b^2x^2 - ax}{b\sqrt{a}\,x^{3/2}\left(\sqrt{ax+b^2x^2} + \sqrt{ax}\right)}$$

$$\lim_{x\to 0^+} \frac{b^2}{b\sqrt{a}\left(\sqrt{a+b^2x} + \sqrt{a}\right)}$$

$$\frac{b}{\sqrt{a}.2\sqrt{a}} \Rightarrow \frac{b}{2a} = 3 \ \Rightarrow \frac{b}{a} = 6$$

**20.** If the term independent of x in the expansion of $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$ is 105, then $a^2$ is equal to :

(1) 4 (2) 9

(3) 6 (4) 2

**Ans. (1)**

**Sol.** $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$

General term $= {}^{10}C_r\left(\sqrt{a}x^2\right)^{10-r}\left(\frac{1}{2x^3}\right)^r$

$20 - 2r - 3r = 0$

$r = 4$

${}^{10}C_4 a^3 \cdot \frac{1}{16} = 105$

$a^3 = 8$

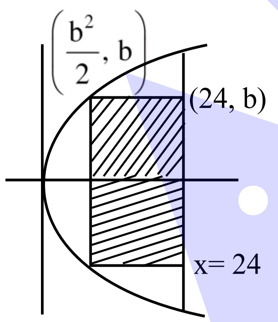
$a^2 = 4$

## SECTION-B

**21.** Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is _________ .

**Ans. (128)**

**Sol.**




$A = 2\left(24 - \frac{b^2}{2}\right).b$

$\frac{dA}{db} = 0 \quad \Rightarrow \quad b = 4$

$A = 2(24 - 8)4$

$= 128$

**22.** If $\alpha = \lim\limits_{x \to 0^+}\left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}\right)$ and $\beta = \lim\limits_{x \to 0}(1 + \sin x)^{\frac{1}{2}\cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then 12 $\log_e(a + b)$ is equal to _________ .

**Ans. (6)**

**Sol.** $\alpha = \lim\limits_{x \to 0^+} e^{\sqrt{x}}\frac{\left(e^{\sqrt{\tan x}-\sqrt{x}} - 1\right)}{\sqrt{\tan x} - \sqrt{x}}$

$= 1$

$\beta = \lim\limits_{x \to 0}(1 + \sin x)^{\frac{1}{2}\cot x}$

$= e^{1/2}$

$x^2 - \left(1 + \sqrt{e}\right) + \sqrt{e} = 0$

$ax^2 + bx - \sqrt{e} = 0$

On comparing

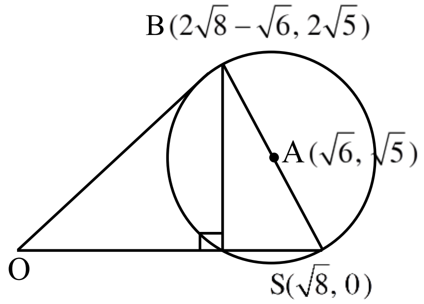
$a = -1, b = \sqrt{e} + 1$

$12\ \ell n(a + b) = 12 \times \frac{1}{2} = 6$

**23.** Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x-axis. Let C be the circle with its centre at $A\left(\sqrt{6}, \sqrt{5}\right)$ and passing through the point S. if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to -

**Ans. (40)**

**Sol.**

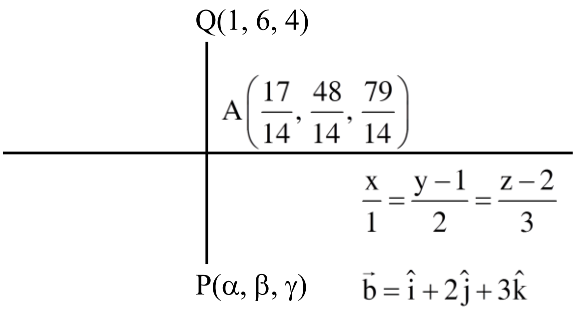



$\text{Area} = \frac{1}{2}(OS)\,h = \frac{1}{2}\sqrt{8}\ 2\sqrt{5} = \sqrt{40}$

**24.** Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to ________ .

**Ans. (11)**

**Sol.**




$A(t, 2t + 1, 3t + 2)$

$\overrightarrow{QA} = (t-1)\hat{i} + (2t-5)\hat{j} + (3t-2)\hat{k}$

$\overrightarrow{QA} \cdot \vec{b} = 0$

$(t - 1) + 2(2t - 5) + 3(3t - 2) = 0$

$14t = 17$

$\alpha = \frac{20}{14} \qquad \beta = \frac{12}{14} \qquad \gamma = \frac{102}{14}$

$2\alpha + \beta + \gamma = \frac{154}{14} = 11$

**25.** An arithmetic progression is written in the following way

$$\begin{array}{ccccccc} & & & 2 & & & \\ & & 5 & & 8 & & \\ & 11 & & 14 & & 17 & \\ 20 & & 23 & & 26 & & 29 \end{array}$$

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The sum of all the terms of the $10^{th}$ row is ______ .

**Ans. (1505)**

**Sol.** 2, 5, 11, 20, …..

General term $= \frac{3n^2 - 3n + 4}{2}$

$T_{10} = \frac{3(100) - 3(10) + 4}{2}$

$= 137$

10 terms with c.d. = 3

$\text{sum} = \frac{10}{2}(2(137) + 9(3))$

$= 1505$

**26.** The number of distinct real roots of the equation $|x + 1|\,|x + 3| - 4\,|x + 2| + 5 = 0$, is _________ .

**Ans. (2)**

**Sol.** $|x + 1|\,|x + 3| - 4|x + 2| + 5 = 0$

**case–1**

$x \le -3$

$(x + 1)(x + 3) + 4(x + 2) + 5 = 0$

$x^2 + 4x + 3 + 4x + 8 + 5 = 0$

$x^2 + 8x + 16 = 0$

$(x + 4)^2 = 0$

$x = -4$

**case-2**

$-3 \le x \le -2$

$-x^2 - 4x - 3 + 4x + 8 + 5 = 0$

$-x^2 + 10 = 0$

$x = \pm\sqrt{10}$

**case-3**

$-2 \le x \le -1$

$-x^2 - 4x - 3 - 4x - 8 + 5 = 0$

$-x^2 - 8x - 6 = 0$

$x^2 + 8x + 6 = 0$

$x = \frac{-8 \pm 2\sqrt{10}}{2} = -4 \pm \sqrt{10}$

**case-4**

$x \ge -1$

$x^2 + 4x + 3 - 4x - 8 + 5 = 0$

$x^2 = 0$

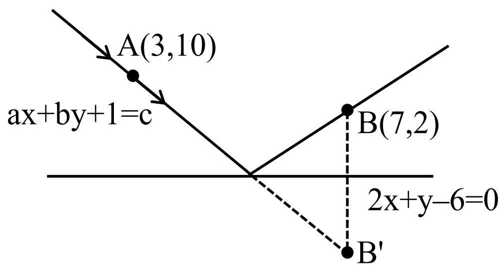
$x = 0$

No. of solution = 2

**27.** Let a ray of light passing through the point (3, 10) reflects on the line $2x + y = 6$ and the reflected ray passes through the point (7, 2). If the equation of the incident ray is $ax + by + 1 = 0$, then $a^2 + b^2 + 3ab$ is equal to_.

**Ans. (1)**

**Sol.**



For B' $\frac{x-7}{2}=\frac{y-2}{1}=-2\left(\frac{14+2-6}{5}\right)$

$\frac{x-7}{2}=\frac{y-2}{1}=-4$

$x=-1 \quad y=-2 \quad B'(-1,-2)$

incident ray AB'

$M_{AB'}=3$

$y+2=3(x+1)$

$3x-y+1=0$

$a=3 \quad b=-1$

$a^2+b^2+3ab=9+1-9=1$

**28.** Let a, b, c $\in$ N and $a<b<c$. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\frac{136}{5}$, respectively. Then $2a+b-c$ is equal to ________ .

**Ans. (33)**

**Sol.** $a, b, c \in N \qquad a<b<c$

$\bar{x}=\text{mean}=\frac{9+25+a+b+c}{5}=18$

$a+b+c=56$

Mean deviation $=\frac{\Sigma|x_i-\bar{x}|}{n}=4$

$=9+7+|18-a|+|18-b|+|18-c|=20$

$=|18-a|+|18-b|+|18-c|=4$

Variance $=\frac{\Sigma|x_i-\bar{x}|^2}{n}=\frac{136}{5}$

$=81+49+|18-a|^2+|18-b|^2+|18-c|^2=136$

$=(18-a)^2+(18-b)^2+(18-c)^2=6$

Possible values $(18-a)^2=1$, $(18-b)^2=1$ $(18-c)^2=4$

$a<b<c$

so $\quad 18-a=1 \quad 18-b=-1 \quad 18-c=-2$

$\quad a=17 \quad b=19 \quad c=20$

$a+b+c=56$

$2a+b-c \quad 34=19-20=33$

**29.** Lei $\alpha|x|=|y|e^{xy-\beta}$, $\alpha, \beta \in N$ be the solution of the differential equation $xdy-ydx+xy(xdy+ydx)=0$, $y(l)=2$. Then $\alpha+\beta$ is equal to _

**Ans. (4)**

**Sol.** $a|x|=|y|\,e^{yx-\beta}$, $a, b \in N$

$xdy-ydx+xy(xdy+ydx)=0$

$\frac{dy}{y}-\frac{dx}{x}+(xdy+ydx)=0$

$\ell n|y|-\ell n|x|+xy=c$

$y(1)=2$

$\ell n|2|-0+2=c$

$c=2+\ell n2$

$\ell n|y|-\ell n|x|+xy=2+\ell n2$

$\ell n|x|=\ell n\left|\frac{y}{2}\right|-2+xy$

$|x|=\left|\frac{y}{2}\right|e^{xy-2}$

$2|x|=|y|e^{xy-2}$

$\alpha=2 \quad \beta=2 \quad \alpha+\beta=4$

**30.** If $\int\frac{1}{\sqrt[5]{(x-1)^4\,(x+3)^6}}dx=A\left(\frac{\alpha x-1}{\beta x+3}\right)^B+C$, where C is the constant of integration, then the value of $\alpha+\beta+20AB$ is ________ .

**Ans. (7)**

**Sol.** $\int\frac{1}{\sqrt[5]{(x-1)^4\,(x+3)^6}}dx=A\left(\frac{\alpha x-1}{\beta x+3}\right)^B+C$

$I=\int\frac{1}{(x-1)^{4/5}\,(x+3)^{6/5}}dx$

$I=\int\frac{1}{\left(\frac{x-1}{x+3}\right)^{4/5}(x+3)^2}dx$

$\left(\frac{x-1}{x+3}\right)=t \Rightarrow \frac{4}{(x+3)^2}dx=dt \qquad t^{-4/5+1}$

$I=\frac{1}{4}\int\frac{1}{t^{4/5}}dt=\frac{1}{4}\frac{t^{1/5}}{1/5}+c$

$I=\frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5}+C$

$A=\frac{5}{4} \qquad \alpha=\beta=1 \qquad B=\frac{1}{5}$

$\alpha+\beta+20AB=2+20\times\frac{5}{4}\times\frac{1}{5}=7$