

FINAL JEE-MAIN EXAMINATION – APRIL, 2024

 (Held On Friday 05th April, 2024)

TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS
TEST PAPER WITH SOLUTION
SECTION-A

1. Let $f: [-1, 2] \rightarrow \mathbb{R}$ be given by
 $f(x) = 2x^2 + x + [x^2] - [x]$, where $[t]$ denotes the greatest integer less than or equal to t . The number of points, where f is not continuous, is :

- (1) 6 (2) 3
 (3) 4 (4) 5

Ans. (3)

Sol. Doubtful points : $-1, 0, 1, \sqrt{2}, \sqrt{3}, 2$

at $x = \sqrt{2}, \sqrt{3}$

$$f(x) = \underbrace{(2x^2 + x - [x])}_{\text{Cont.}} + \underbrace{[x^2]}_{\text{Cont.}} = \text{Discount}$$

at $x = -1$:

$$\text{RHL} \Rightarrow f(x) = (2 - 1 - (-1)) + 0 = 2 \quad \left. \begin{array}{l} \\ f(-1) = 2 - 1 - (-1) + 1 = 3 \end{array} \right\} \text{Dis.}$$

at $x = 2$:

$$\text{LHL} \Rightarrow f(x) = 8 + 2 - 1 + 3 = 12 \quad \left. \begin{array}{l} \\ f(2) = 8 + 2 - 2 + 4 = 12 \end{array} \right\} \text{Cont.}$$

at $x = 0$:

$$\text{LHL} \Rightarrow 0 + 0 - (-1) + 0 = 1 \quad \left. \begin{array}{l} \\ f(0) = 0 \end{array} \right\} \text{Dis.}$$

at $x = 1$

$$\text{LHL} \Rightarrow 2 + 1 - 0 + 0 = 3 \quad \left. \begin{array}{l} \\ f(1) = 3 - 1 + 1 = 3 \end{array} \right\} \text{Cont.}$$

$$\text{RHL} \Rightarrow 2 + 1 - 1 + 1 = 3$$

2. The differential equation of the family of circles passing the origin and having center at the line $y = x$ is :

- (1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$
 (2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$
 (3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$
 (4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

Ans. (3)

Sol. $C \equiv x^2 + y^2 + gx + gy = 0 \quad \dots(1)$

$$2x + 2yy' + g + gy' = 0$$

$$g = -\left(\frac{2x + 2yy'}{1 + y'}\right)$$

Put in (1)

$$x^2 + y^2 - \left(\frac{2x + 2yy'}{1 + y'}\right)(x + y) = 0$$

$$(x^2 - y^2 - 2xy)y' = x^2 - y^2 + 2xy$$

3. Let $S_1 = \{z \in \mathbb{C} : |z| \leq 5\}$,

$$S_2 = \left\{z \in \mathbb{C} : \text{Im}\left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i}\right) \geq 0\right\} \text{ and}$$

$S_3 = \{z \in \mathbb{C} : \text{Re}(z) \geq 0\}$. Then the area of region

$S_1 \cap S_2 \cap S_3$ is

- (1) $\frac{125\pi}{6}$ (2) $\frac{125\pi}{24}$
 (3) $\frac{125\pi}{4}$ (4) $\frac{125\pi}{12}$

Ans. (4)

Sol. $S_1 : x^2 + y^2 \leq 25 \quad \dots(1)$

$$S_2 : \text{Im of } \frac{z + (1 - \sqrt{3}i)}{(1 - \sqrt{3}i)} \geq 0$$

$$\text{Im of } \left(\frac{x + iy}{1 - \sqrt{3}i} + 1\right) \geq 0$$

$$\text{Im of } \left(\frac{(x + iy)(1 + \sqrt{3}i)}{4}\right) \geq 0$$

$$\Rightarrow \sqrt{3}x + y \geq 0 \quad \dots(2)$$

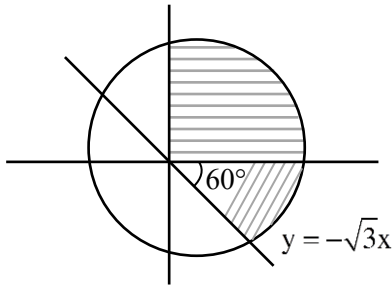
$$S_3 : x \geq 0 \quad \dots(3)$$

$$\text{Area} = \frac{5}{12}(\pi(5)^2)$$



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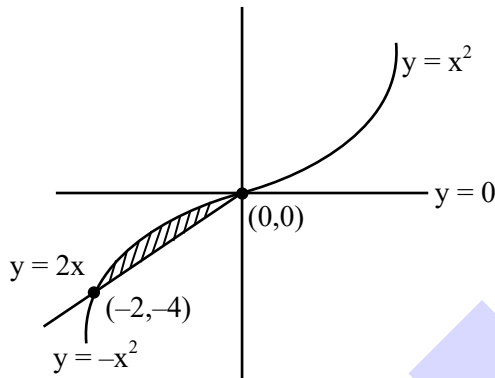


4. The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is :

- (1) $\frac{8}{3}$ (2) $\frac{2}{3}$
(3) 1 (4) $\frac{4}{3}$

Ans. (4)

Sol.



$$A = \int_{-2}^0 -x^2 - 2x = \frac{4}{3}$$

5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is :

- (1) OBBHJ (2) HBBJO
(3) OBBJH (4) JBBOH

Ans. (3)

Sol. B B H J O

[B] _____ $4! = 24$

[H] _____ $\frac{4!}{2!} = 12$

[J] _____ $\frac{4!}{2!} = 12$

O B B H J

O B B J H \rightarrow 50th rank

6. Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$

and \vec{c} be three vectors such that

$$(\vec{c} + \hat{i}) \times (\vec{a} + \vec{b} + \hat{i}) = \vec{a} \times (\vec{c} + \hat{i}) \quad \vec{a} \cdot \vec{c} = -29,$$

then $\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k})$ is equal to :

- (1) 10 (2) 5
(3) 15 (4) 12

Ans. (2)

Sol. Let's assume $\vec{v} = \vec{a} + \vec{b} + \hat{i}$

$$= 5\hat{i} + 3\hat{j} + \hat{k}$$

$$\text{and } \vec{c} + \hat{i} = \vec{p}$$

So,

$$\vec{p} \times \vec{v} = \vec{a} \times \vec{p}$$

$$\vec{p} \times \vec{v} + \vec{p} \times \vec{a} = \vec{0}$$

$$\vec{p} \times (\vec{v} + \vec{a}) = \vec{0}$$

$$\Rightarrow \vec{p} = \lambda(\vec{v} + \vec{a})$$

$$\vec{c} + \hat{i} = \lambda(7\hat{i} + 8\hat{j})$$

$$\vec{a} \cdot \vec{c} + \vec{a} \cdot \hat{i} = \lambda \vec{a} \cdot (7\hat{i} + 8\hat{j})$$

$$-29 + 2 = \lambda(14 + 40)$$

$$\lambda = -\frac{1}{2}$$

$$\begin{aligned} \vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) + \hat{i} \cdot (-2\hat{i} + \hat{j} + \hat{k}) &= \lambda(7\hat{i} + 8\hat{j}) \cdot (-2\hat{i} + \hat{j} + \hat{k}) \\ &= -\frac{1}{2}(-14 + 8) + 2 = 5 \end{aligned}$$

7. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$

and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between

the vectors \vec{b} and \vec{c} , then the minimum value of

$27|\vec{c} - \vec{a}|^2$ is equal to :

- (1) 110 (2) 105
(3) 124 (4) 121

Ans. (3)



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Sol. $|\vec{c} - \vec{a}| = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c}$

$$= |\vec{c}|^2 + 4 - 0$$

$$\therefore \vec{a} = \vec{b} \times \vec{c}$$

$$|\vec{a}| = |\vec{b} \times \vec{c}|$$

$$2 = 3|\vec{c}|\sin\alpha$$

$$|\vec{c}| = \frac{2}{3} \operatorname{cosec}\alpha \quad \alpha \in \left[0, \frac{\pi}{3}\right]$$

$$|\vec{c}|_{\min} = \frac{2}{3} \times \frac{2}{\sqrt{3}} \quad \operatorname{cosec}\alpha \in \left[\frac{2}{\sqrt{3}}, \infty\right)$$

$$\Rightarrow 27|\vec{c} - \vec{a}|_{\min}^2 = 27\left(\frac{16}{27} + 4\right) = 124$$

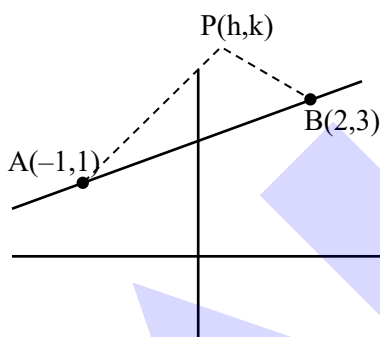
8. Let A(-1, 1) and B(2, 3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is $ax + by = 15$, then $5a + 2b$ is :

(1) $-\frac{12}{5}$ (2) $-\frac{6}{5}$

(3) 4 (4) 6

Ans. (1)

Sol.



$$\frac{1}{2} \begin{vmatrix} h & k & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 10$$

$$-2x + 3y = 25$$

$$-\frac{6}{5}x + \frac{9}{5}y = 15$$

$$a = -\frac{6}{5}, b = \frac{9}{5}$$

$$5a = -6, 2b = \frac{18}{5}$$

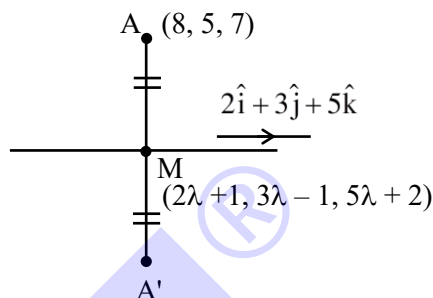
9. Let (α, β, γ) be the image of the point (8, 5, 7) in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is equal to

(1) 16 (2) 18

(3) 14 (4) 20

Ans. (3)

Sol.



$$\vec{AM} \cdot (2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$$

$$(2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$$

$$38\lambda = 57$$

$$\lambda = \frac{3}{2}$$

$$M\left(4, \frac{7}{2}, \frac{19}{2}\right)$$

$$A'(0, 2, 12)$$

10. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}$, $x \neq 0$, is $\alpha \times 2^8 \times \sqrt[5]{3}$, then 25α is equal to :

(1) 639 (2) 724

(3) 693 (4) 742

Ans. (3)

Sol. $T_{r+1} = {}^{12}C_r \left(\frac{3^{1/5}}{x}\right)^{12-r} \left(\frac{2x}{5^{1/3}}\right)^r$

$$T_{r+1} = \frac{{}^{12}C_r (3)^{\frac{12-r}{5}} (2)^r (x)^{2r-12}}{(5)^{r/3}}$$

$$r = 6$$

$$T_7 = \frac{{}^{12}C_6 (3)^{6/5} (2)^6}{5^2} = \left(\frac{9 \times 11 \times 7}{25}\right) 2^8 \cdot 3^{1/5}$$

$$25\alpha = 693$$



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11. Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined as : $f(x) = |x - 1|$ and

$$g(x) = \begin{cases} e^x, & x \geq 0 \\ x + 1, & x < 0 \end{cases}. \text{ Then the function } f(g(x)) \text{ is}$$

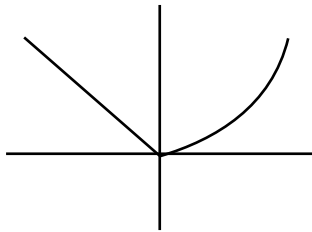
- (1) neither one-one nor onto.
(2) one-one but not onto.
(3) both one-one and onto.
(4) onto but not one-one.

Ans. (1)

Sol. $f(g(x)) = |g(x) - 1|$

$$\text{fog} \begin{cases} |e^x - 1| & x \geq 0 \\ |x + 1 - 1| & x < 0 \end{cases}$$

$$\text{fog} \begin{cases} e^x - 1 & x \geq 0 \\ -x & x < 0 \end{cases}$$



12. Let the circle $C_1 : x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at $(-1, 0)$ and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

- (1) 2 (2) 1
(3) 6 (4) 4

Ans. (1)

Sol. $S_1 : x^2 + y^2 - 2x - 2y + 1 = 0$

$$S_2 : x^2 + y^2 + 2x - 3 = 0$$

$$\text{Common chord} = S_1 - S_2 = 0$$

$$-4x - 2y + 4 = 0$$

$$2x + y = 2 \Rightarrow P(0, 2)$$

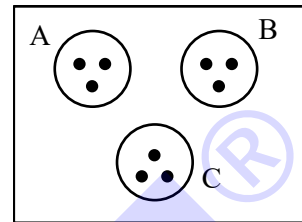
$$d_{(c,p)}^2 = (1 - 0)^2 + (2 - 1)^2 = 2$$

13. Let the set $S = \{2, 4, 8, 16, \dots, 512\}$ be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to :

- (1) 1680 (2) 1520
(3) 1710 (4) 1640

Ans. (1)

Sol.



$$\frac{9!}{(3!3!3!)3!} \times 3!$$

14. The values of m, n, for which the system of equations

$$\begin{aligned} x + y + z &= 4, \\ 2x + 5y + 5z &= 17, \\ x + 2y + mz &= n \end{aligned}$$

has infinitely many solutions, satisfy the equation :

- (1) $m^2 + n^2 - m - n = 46$
(2) $m^2 + n^2 + m + n = 64$
(3) $m^2 + n^2 + mn = 68$
(4) $m^2 + n^2 - mn = 39$

Ans. (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 2$

$$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 17 \\ 1 & 2 & n \end{vmatrix} = 0 \Rightarrow n = 7$$

15. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set $\{1, 2, 3, 4, 5, 6\}$. If the probability of this equation having one real root bigger than the other is p, then $216p$ equals :

- (1) 57 (2) 38
(3) 19 (4) 76

Ans. (2)



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Sol. $D > 0$

$$b^2 > 4ac$$

$$b = 3 : (a, c) = (1, 1)(1, 2)(2, 1)$$

$$b = 4 : (a, c) = (1, 1)(1, 2)(2, 1)(1, 3)(3, 1)$$

$$b = 5 : (a, c) = (1, 1)(1, 2)(2, 1)(1, 3)(3, 1)(1, 4)(4, 1)(1, 5)(5, 1)(1, 6)(6, 1)(2, 3)(3, 2)(2, 2)$$

$$b = 6 : (a, c) = (1, 1)(1, 2)(2, 1)(1, 3)(3, 1)(1, 4)(4, 1)(1, 5)(5, 1)(1, 6)(6, 1)(2, 3)(3, 2)(2, 4)(4, 2)(2, 2)$$

fav. cases = 38

$$\text{Prob.} : \frac{38}{6 \times 6 \times 6}$$

16. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies :

$$(1) r = 1$$

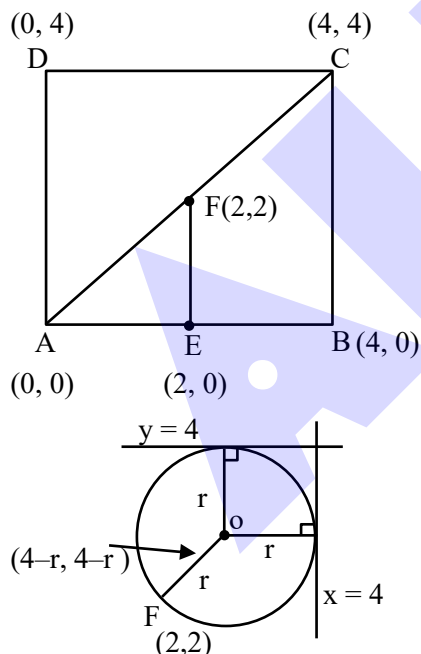
$$(2) r^2 - 8r + 8 = 0$$

$$(3) 2r^2 - 4r + 1 = 0$$

$$(4) 2r^2 - 8r + 7 = 0$$

Ans. (2)

Sol.



$$OF^2 = r^2$$

$$(2 - r)^2 + (2 - r)^2 = r^2$$

$$r^2 - 8r + 8 = 0$$

17. Let $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$, $m, n > 0$. If

$$\int_0^1 (1-x^{10})^{20} dx = a \times \beta(b, c), \text{ then } 100(a + b + c) \text{ equals } \underline{\hspace{2cm}}.$$

$$(1) 1021$$

$$(2) 1120$$

$$(3) 2012$$

$$(4) 2120$$

Ans. (4)

Sol. $I = \int_0^1 1 \cdot (1-x^{10})^{20} dx$

$$x^{10} = t$$

$$x = t^{1/10}$$

$$dx = \frac{1}{10} (t)^{-9/10} dt$$

$$I = \int_0^1 (1-t)^{20} \frac{1}{10} (t)^{-9/10} dt$$

$$I = \frac{1}{10} \int_0^1 t^{-9/10} (1-t)^{20} dt$$

$$a = \frac{1}{10}$$

$$b = \frac{1}{10}$$

$$c = 21$$

18. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$.

$$\text{If } B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix} \text{ is the matrix of cofactors}$$

of the elements of A, then $\det(AB)$ is equal to :

$$(1) 343$$

$$(2) 125$$

$$(3) 64$$

$$(4) 216$$

Ans. (4)

- Sol. Equating co-factor fo A_{21}

$$(2\alpha^2 - 3\alpha) = \alpha$$

$$\alpha = 0, 2 \text{ (accept)}$$

$$\text{Now, } 2\alpha^2 - \alpha\beta = 3\alpha$$

$$\alpha = 2 \quad \beta = 1$$

$$|AB| = |A \text{ cof}(A)| = |A|^3$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix} = 6 - 2(9) + 3(6) = 6$$



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19. If $y(\theta) = \frac{2\cos\theta + \cos 2\theta}{\cos 3\theta + 4\cos 2\theta + 5\cos\theta + 2}$,
then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to:

- (1) $\frac{3}{2}$ (2) 1
 (3) $\frac{1}{2}$ (4) 2

Ans. (4)

Sol. $y = \frac{2\cos\theta + 2\cos^2\theta - 1}{4\cos^3\theta - 3\cos\theta + 8\cos^2\theta - 4 + 5\cos\theta + 2}$

$$y = \frac{(2\cos^2\theta + 2\cos\theta - 1)}{(2\cos^2\theta + 2\cos\theta - 1)(2\cos\theta + 2)}$$

$$y = \frac{1}{2} \left(\frac{1}{1 + \cos\theta} \right)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = \frac{1}{2}$$

$$y' = \frac{1}{2} \left(\frac{-1}{(1 + \cos\theta)^2} \times (-\sin\theta) \right)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = \frac{1}{2}$$

$$y'' = \frac{1}{2} \left[\frac{\cos\theta(1 + \cos\theta)^2 - \sin\theta(2)(1 + \cos\theta)(-\sin\theta)}{(1 + \cos\theta)^4} \right]$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

$$y = 1$$

20. For $x \geq 0$, the least value of K, for which $4^{1+x} + 4^{1-x}$, $\frac{K}{2}$, $16^x + 16^{-x}$ are three consecutive terms of an A.P. is equal to :

- (1) 10 (2) 4
 (3) 8 (4) 16

Ans. (1)

Sol. $k = 4 \left(4^x + \frac{1}{4^x} \right) + \left(4^{2x} + \frac{1}{4^{2x}} \right)$

$$\geq 2 \quad \geq 2$$

$$k \geq 10$$

SECTION-B

21. Let the mean and the standard deviation of the probability distribution

X	α	1	0	-3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ , respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to _____.

Ans. (5)

Sol. $\frac{1}{3} + k + \frac{1}{6} + \frac{1}{4} = 1 \Rightarrow k = \frac{1}{4}$

$$\mu = \frac{\alpha}{3} + \frac{1}{4} - \frac{3}{4}$$

$$\mu = \frac{\alpha}{3} - \frac{1}{2}$$

$$\sigma = \sqrt{\left(\alpha^2 \frac{1}{3} + \frac{1}{4} + 9 \frac{1}{4} \right) - \left(\frac{\alpha}{3} - \frac{1}{2} \right)^2}$$

$$\sigma = \sqrt{\frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4}}$$

$$\sigma = \mu + 2$$

$$\sigma^2 = (\mu + 2)^2 \Rightarrow \frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4} = \frac{\alpha^2}{9} + \frac{9}{4} + \alpha$$

$$\frac{\alpha^2}{9} - \frac{2\alpha}{3} = 0$$

$$\alpha = 0, \text{ (reject) or } \alpha = 6$$

$$(\because x = 0 \text{ is already given})$$

$$\Rightarrow \sigma + \mu = 2\mu + 2$$

$$= 5$$

22. Let $y = y(x)$ be the solution of the differential

equation $\frac{dy}{dx} + \frac{2x}{(1+x^2)^2} y = x e^{\frac{1}{(1+x^2)}}$; $y(0) = 0$.

Then the area enclosed by the curve

$$f(x) = y(x) e^{-\frac{1}{(1+x^2)}}$$
 and the line $y - x = 4$ is _____.

Ans. (18)



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Sol. $IF = e^{\int \frac{2x}{(1+x^2)^2} dx} = e^{\frac{-1}{1+x^2}}$

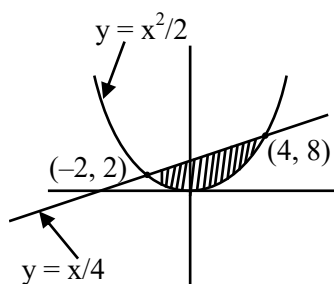
$$y \cdot e^{\frac{-1}{1+x^2}} = \int x \cdot e^{\frac{1}{1+x^2}} \cdot e^{\frac{-1}{1+x^2}} dx$$

$$y \cdot e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + c$$

$$(0, 0) \Rightarrow \boxed{C=0}$$

$$y(x) = \frac{x^2}{2} e^{\frac{1}{1+x^2}}$$

$$f(x) = \frac{x^2}{2}$$

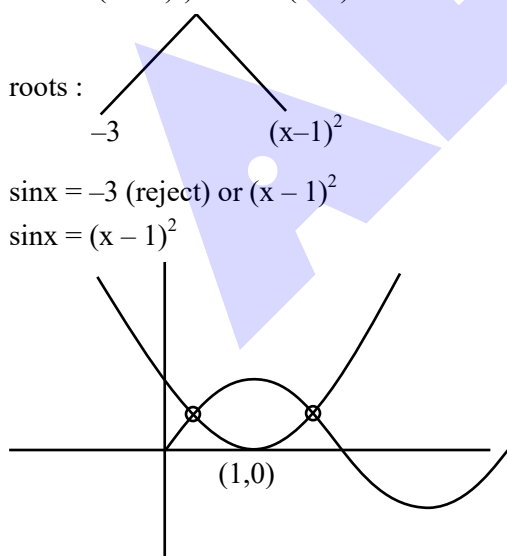


$$A = \int_{-2}^4 (x+4) - \frac{x^2}{2} dx = 18$$

- 23.** The number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x-1)^2 = 0$, where $-\pi \leq x \leq \pi$, is

Ans. (2)

Sol. $\sin^2 x - (x^2 - 2x - 2)\sin x - 3(x-1)^2 = 0$
 $\sin^2 x - (x-1)^2 \sin x - 3(x-1)^2 = 0$

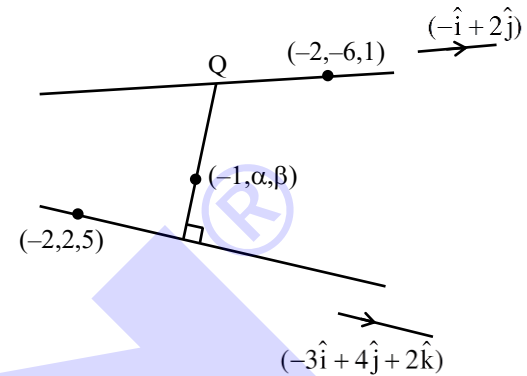


- 24.** Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$.

Then $(\alpha - \beta)^2$ is equal to ____.

Ans. (25)

Sol.



$$P(-3\lambda - 2, 4\lambda + 2, 2\lambda + 5)$$

$$Q(-\mu - 2, 2\mu - 6, 1)$$

$$\text{DRS of } PQ = (3\lambda - \mu, 2\mu - 4\lambda - 8, -2\lambda - 4)$$

$$\text{DRS of } PQ = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$$

$$= (4\hat{i} + 2\hat{j} + 2\hat{k})$$

OR

$$(2, 1, 1)$$

$$\frac{3\lambda - \mu}{2} = \frac{2\mu - 4\lambda - 8}{1} = \frac{-2\lambda - 4}{1}$$

$$\Rightarrow \mu = \lambda + 2 \text{ \& } 7\lambda = \mu - 8$$

$$\boxed{\lambda = -1} \quad \boxed{\mu = 1}$$

$$Q : (-3, -4, 1)$$

$$L_{PQ} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$$

$$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha+4}{1} = \frac{\beta-1}{1}$$

$$\Rightarrow \alpha = -3, \beta = 2$$

$$(\alpha - \beta)^2 = 25$$



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25. If

$$1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \frac{49-20\sqrt{6}}{180} + \dots$$

upto $\infty = 2\left(\sqrt{\frac{b}{a}} + 1\right) \log_e\left(\frac{a}{b}\right)$, where a and b are

integers with $\gcd(a, b) = 1$, then $11a + 18b$ is equal to _____.

Ans. (76)

Sol. $S = 1 + \frac{x}{2\sqrt{3}} + \frac{x^2}{18} + \frac{x^3}{36\sqrt{3}} + \frac{x^4}{180} + \dots \infty$

Put $\frac{x}{\sqrt{3}} = t$, where $x = \sqrt{3} - \sqrt{2}$

$$S = 1 + \frac{t}{2} + \frac{t^2}{6} + \frac{t^3}{12} + \frac{t^4}{20} + \dots$$

$$S = 1 + t\left(1 - \frac{1}{2}\right) + t^2\left(\frac{1}{2} - \frac{1}{3}\right) + t^3\left(\frac{1}{3} - \frac{1}{4}\right) + t^4\left(\frac{1}{4} - \frac{1}{5}\right) + \dots$$

$$S = \left(1 + t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{4} + \dots\right) - \left(\frac{t}{2} + \frac{t^2}{3} + \frac{t^3}{4} + \frac{t^4}{5} + \dots\right)$$

$$S = \left(t + \frac{t^2}{2} + \dots\right) - \frac{1}{t}\left(t + \frac{t^2}{2} + \frac{t^3}{3} + \dots\right) + 2$$

$$S = 2 + \left(1 - \frac{1}{t}\right)(-\log(1-t)) = \left(\frac{1}{t} - 1\right) \log(1-t) + 2$$

$$S = 2 + \left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} - 1\right) \log\left(1 - \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$$

$$S = 2 + \left(\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) \log_e \frac{\sqrt{2}}{\sqrt{3}}$$

$$S = 2 + \frac{(\sqrt{6}+2)}{2} \log_e \frac{2}{3} = 2 + \left(\sqrt{\frac{3}{2}} + 1\right) \log_e \frac{2}{3}$$

$$a = 2, b = 3$$

$$11a + 18b = 11 \times 2 + 18 \times 3 = 76$$

26. Let $a > 0$ be a root of the equation $2x^2 + x - 2 = 0$.

If $\lim_{x \rightarrow \frac{1}{a}} \frac{16(1 - \cos(2 + x - 2x^2))}{(1 - ax^2)} = \alpha + \beta\sqrt{17}$, where

$\alpha, \beta \in \mathbb{Z}$ then $\alpha + \beta$ is equal to _____.

Ans. (170)

Sol. $2x^2 + x - 2 = 0$

$$\lim_{x \rightarrow \frac{1}{a}} \frac{16 \cdot \frac{1 - \cos\left(2 + x - \frac{1}{a}\right)\left(x - \frac{1}{b}\right)}{4\left(x - \frac{1}{b}\right)^2} \times \frac{4\left(x - \frac{1}{b}\right)^2}{a^2\left(x - \frac{1}{a}\right)^2}}{1} = \frac{16 \times \frac{2}{a^2} \left(\frac{1}{a} - \frac{1}{b}\right)^2}{\frac{32}{a^2} \left(\frac{17}{4}\right) = \frac{17.8}{a^2} = \frac{17 \times 8 \times 16}{(-1 + \sqrt{117})^2}}$$

$$= \frac{136.16}{18.2\sqrt{7}} \times \frac{18 + 2\sqrt{7}}{18 + 2\sqrt{7}} = \frac{136}{256} (18 + 2\sqrt{7}) \cdot 16 = 153 + 17\sqrt{17} = \alpha + \beta\sqrt{17}$$

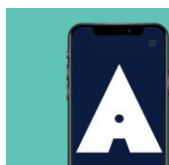
$$\alpha + \beta = 153 + 17 = 170$$

27. If $f(t) = \int_0^{\pi} \frac{2x dx}{1 - \cos^2 t \sin^2 x}$, $0 < t < \pi$, then the value

of $\int_0^{\frac{\pi}{2}} \frac{\pi^2 dt}{f(t)}$ equals _____.

Ans. (1)

Sol. $f(t) = \int_0^{\pi} \frac{2x}{1 - \cos^2 t \sin^2 x} dx$ (1)



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$$= 2 \int_0^{\pi} \frac{(\pi - x) dx}{1 - \cos^2 t \sin^2 x} \quad \dots(2)$$

$$2f(t) = 2 \int_0^{\pi} \frac{\pi}{1 - \cos^2 t \sin^2 x} dx$$

$$f(t) = \int_0^{\pi} \frac{\pi}{1 - \cos^2 t \sin^2 x} dx$$

divide & by $\cos^2 x$

$$f(t) = \pi \int_0^{\pi} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$$f(t) = 2\pi \int_0^{\pi/2} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$$\tan x = z$$

$$\sec^2 x dx = dz$$

$$f(t) = 2\pi \int_0^{\infty} \frac{dz}{1 + \sin^2 t \cdot z^2}$$

$$= \frac{\pi^2}{\sin t}$$

$$\text{Then } \int_0^{\pi/2} \frac{\pi^2}{f(t)} dt$$

$$= \int_0^{\pi/2} \sin t dt$$

$$= 1$$

28. Let the maximum and minimum values of $(\sqrt{8x - x^2 - 12} - 4)^2 + (x - 7)^2$, $x \in \mathbb{R}$ be M and m respectively. Then $M^2 - m^2$ is equal to _____.

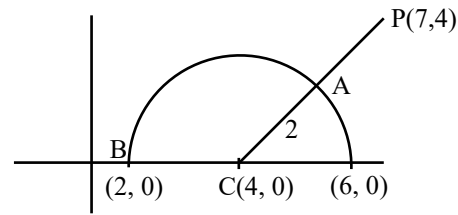
Ans. (1600)

Sol. $(x - 7)^2 + (y - 4)^2$

$$y = \sqrt{8x - x^2 - 12}$$

$$y^2 = -(x - 4)^2 + 16 - 12$$

$$(x - 4)^2 + y^2 = 4$$



$$m = 9$$

$$M = 41$$

$$M^2 - m^2 = 41^2 - 9^2 = 1600$$

29. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P . The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is _____.

Ans. (10)

Sol. $y^2 = 4(x - 9)$

$$\text{slope of tangent} = \frac{-1}{2}$$

$$\text{Point of contact } P \left(9 + \frac{1}{\left(\frac{-1}{2} \right)^2}, \frac{2 \times 1}{\frac{-1}{2}} \right)$$

$$P(13, -4)$$

$$\text{center of circle } C(7, 4)$$

$$\text{distance } CP = \sqrt{(13 - 7)^2 + (-4 - 4)^2} = 10$$

30. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is _____.

Ans. (3)

30. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is _____.

Allen Ans. (3)

Sol. Case I : $x \geq -5$

$$x^2 + 5x + 2x + 12 = 0$$

$$x^2 + 7x + 12 = 0$$

$$x = -3, -4$$

Case II : $-7 < x < -5$

$$-x^2 - 5x + 2x + 14 - 2 = 0$$



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$$-x^2 - 3x + 12 = 0$$

$$x = \frac{-3 \pm \sqrt{9 + 48}}{2}$$

$$= \frac{-3 \pm \sqrt{57}}{2}$$

$$x = \frac{-3 - \sqrt{57}}{2}, \frac{-3 + \sqrt{57}}{2} \text{ (rejected)}$$

Case III : $x \leq -7$

$$-x^2 - 5x - 2x - 14 - 2 = 0$$

$$x^2 + 7x + 16 = 0$$

$$D = 49 - 64 < 0$$

No solutions

No. of solutions = 3

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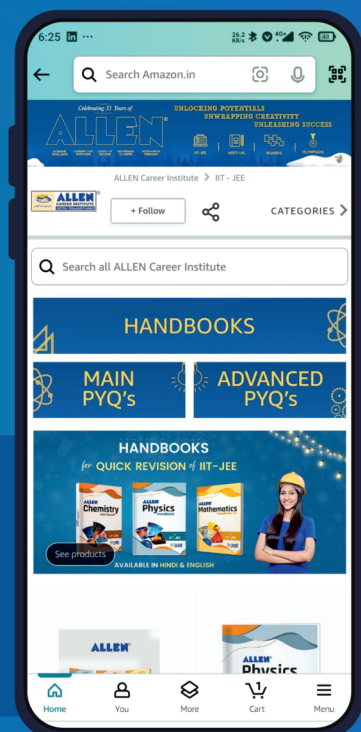
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