FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. Let the line L intersect the lines

$x - 2 = -y = z - 1$, $2(x + 1) = 2(y - 1) = z + 1$

and be parallel to the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$.

Then which of the following points lies on L ?

(1) $\left(-\frac{1}{3}, 1, 1\right)$ (2) $\left(-\frac{1}{3}, 1, -1\right)$

(3) $\left(-\frac{1}{3}, -1, -1\right)$ (4) $\left(-\frac{1}{3}, -1, 1\right)$

Ans. (2)

2. The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to :

(1) $\frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (2) $\frac{1}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(3) $\frac{1}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (4) $\frac{2}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Ans. (1)

3. The solution curve, of the differential equation

$2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx}$, passing through the point

(0, 1) is a conic, whose vertex lies on the line :

(1) $2x + 3y = 9$ (2) $2x + 3y = -9$

(3) $2x + 3y = -6$ (4) $2x + 3y = 6$

Ans. (1)

4. A ray of light coming from the point P (1, 2) gets reflected from the point Q on the x-axis and then passes through the point R (4, 3). If the point S (h, k) is such that PQRS is a parallelogram, then hk^2 is equal to :

(1) 80 (2) 90

(3) 60 (4) 70

Ans. (4)

5. Let $\lambda, \mu \in \mathbb{R}$. If the system of equations

$3x + 5y + \lambda z = 3$

$7x + 11y - 9z = 2$

$97x + 155y - 189z = \mu$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to :

(1) 25 (2) 24

(3) 27 (4) 22

Ans. (1)

6. The coefficient of x^{70} in $x^2(1 + x)^{98} + x^3(1 + x)^{97} + x^4(1 + x)^{96} + \dots + x^{54}(1 + x)^{46}$ is $^{99}C_p - {}^{46}C_q$.

Then a possible value to p + q is :

(1) 55 (2) 61

(3) 68 (4) 83

Ans. (4)

7. Let

$$\int \frac{2 - \tan x}{3 + \tan x}dx = \frac{1}{2}\left(\alpha x + \log_e|\beta \sin x + \gamma \cos x|\right) + C$$

, where C is the constant of integration.

Then $\alpha + \frac{\gamma}{\beta}$ is equal to :

(1) 3 (2) 1

(3) 4 (4) 7

Ans. (3)

8. A variable line L passes through the point (3, 5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is :

(1) 30 (2) 25

(3) 40 (4) 35

Ans. (1)

9. Let

$$\left|\cos\theta\cos(60-\theta)\cos(60-\theta)\right| \le \frac{1}{8}, \theta \in [0, 2\pi]$$

Then, the sum of all $\theta \in [0, 2\pi]$, where cos 3θ attains its maximum value, is :

(1) 9π (2) 18 π

(3) 6 π (4) 15 π

Ans. (3)

10. Let $\overrightarrow{OA} = 2\vec{a}, \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$ and $\overrightarrow{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral OABC is equal to :

(1) 38 (2) 40

(3) 32 (4) 35

Ans. (4)

11. If the domain of the function

$f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is R – (α, β)

then 12αβ is equal to :

(1) 36 (2) 24

(3) 40 (4) 32

Ans. (4)

12. If the sum of series

$$\frac{1}{1\cdot(1+d)} + \frac{1}{(1+d)(1+2d)} + \ldots\ldots + \frac{1}{(1+9d)(1+10d)}$$

is equal to 5, then 50d is equal to :

(1) 20 (2) 5

(3) 15 (4) 10

Ans. (2)

13. Let $f(x) = ax^3 + bx^2 + ex + 41$ be such that

f(1) = 40, f'(1) = 2 and f''(1) = 4.

Then $a^2 + b^2 + c^2$ is equal to :

(1) 62 (2) 73

(3) 54 (4) 51

Ans. (4)

14. Let a circle passing through (2, 0) have its centre at the point (h, k). Let (x_c, y_c) be the point of intersection of the lines 3x + 5y = 1 and (2 + c) x + $5c^2y$ = 1. If $h = \lim_{c\to1} x_c$ and $k = \lim_{c\to1} y_c$, then the equation of the circle is :

(1) $25x^2 + 25y^2 - 20x + 2y - 60 = 0$

(2) $5x^2 + 5y^2 - 4x - 2y - 12 = 0$

(3) $25x^2 + 25y^2 - 2x + 2y - 60 = 0$

(4) $5x^2 + 5y^2 - 4x + 2y - 12 = 0$

Ans. (1)

15. The shortest distance between the line

$$\frac{x-3}{4} = \frac{y+7}{-11} = \frac{z-1}{5} \text{ and } \frac{x-5}{3} = \frac{y-9}{-6} = \frac{z+2}{1}$$

is :

(1) $\frac{187}{\sqrt{563}}$ (2) $\frac{178}{\sqrt{563}}$

(3) $\frac{185}{\sqrt{563}}$ (4) $\frac{179}{\sqrt{563}}$

Ans. (1)

16. The frequency distribution of the age of students in a class of 40 students is given below.

Age	15	16	17	18	19	20
No. of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then 4x + 5y is equal to :

(1) 43 (2) 44

(3) 47 (4) 46

Ans. (2)

17. The solution of the differential equation

$(x^2 + y^2)dx - 5xy\,dy = 0$, y(1) = 0, is :

(1) $\left|x^2 - 4y^2\right|^5 = x^2$ (2) $\left|x^2 - 2y^2\right|^6 = x$

(3) $\left|x^2 - 4y^2\right|^6 = x$ (4) $\left|x^2 - 2y^2\right|^5 = x^2$

Ans. (1)

18. Let three vectors $\vec{a} = \alpha\hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 5\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ from a triangle such that $\vec{c} = \vec{a} - \vec{b}$ and the area of the triangle is $5\sqrt{6}$. if α is a positive real number, then $|\vec{c}|^2$ is :

(1) 16 (2) 14

(3) 12 (4) 10

Ans. (2)

19. Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is :

(1) $x^2 - 190x + 9466 = 0$

(2) $x^2 - 195x + 9466 = 0$

(3) $x^2 - 195x + 9506 = 0$

(4) $x^2 - 180x + 9506 = 0$

Ans. (3)

20. Let $f(x) = x^2 + 9$, $g(x) = \frac{x}{x-9}$ and a = fog(10), b = gof(3). If e and l denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$, then $8e^2 + l^2$ is equal to.

(1) 16 (2) 8

(3) 6 (4) 12

Ans. (2)

SECTION-B

21. Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked 1, 2, 3, 4. If the probability that $ax^2 + bx + c = 0$ has all real roots is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to ________.

Ans. (19)

22. The sum of the square of the modulus of the elements in the set $\{z = a + ib : a, b \in Z, z \in C, |z-1| \le 1, |z-5| \le |z-5i|\}$ is ________.

Ans. (9)

23. Let the set of all positive values of λ, for which the point of local minimum of the function $(1 + x(\lambda^2 - x^2))$ satisfies $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$, be (α, β). Then $\alpha^2 + \beta^2$ is equal to ________.

Ans. (39)

24. Let

$$\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}} - \frac{2n}{(n^2+1)\sqrt{n^4+1}} + \frac{n}{\sqrt{n^4+16}} - \frac{8n}{(n^2+4)\sqrt{n^4+16}} + \ldots\ldots + \frac{n}{\sqrt{n^4+n^4}} - \frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right)$$

be $\frac{\pi}{k}$, using only the principal values of the inverse trigonometric functions. Then k^2 is equal to ______.

Ans. (32)

25. The remainder when 428^{2024} is divided by 21 is ________.

Ans. (1)

26. Lef $f:(0, \pi) \to R$ be a function given by

$$f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0 < x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2} < x < \pi \end{cases}$$

Where $a, b \in Z$. If f is continuous at $x = \frac{\pi}{2}$, then $a^2 + b^2$ is equal to ________.

Ans. (81)

27. Let A be a non-singular matrix of order 3. If $\det(3\text{adj}(2\text{adj}((\det A)A))) = 3^{-13} \cdot 2^{-10}$ and $\det(3\text{adj}(2A)) = 2^m \cdot 3^n$, then $|3m+2n|$ is equal to ________.

Ans. (14)

28. Let the centre of a circle, passing through the point (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$, be (h, k). Then for all possible values of the coordinates of the centre (h, k), $4(h^2 + k^2)$ is equal to ________.

Ans. (9)

29. If a function f satisfies f(m + n) = f(m) + f(n) for all m, n $\in$ N and f(1) = 1, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda+k) \le (2022)^2$ is equal to ________.

Ans. (1010)

30. Let A = {2, 3, 6, 7} and B = {4, 5, 6, 8}. Let R be a relation defined on A × B by (a_1, b_1) R (a_2, b_2) is and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is ________.

Ans. (25)