FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. $\lim\limits_{x\to 0}\dfrac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to :

(1) e (2) $\dfrac{-2}{e}$

(3) 0 (4) $e-e^2$

Ans. (1)

2. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $\left(\dfrac{11}{3},\dfrac{11}{3},\dfrac{19}{3}\right)$ from the line L along the line $\dfrac{3x-11}{2}=\dfrac{3y-11}{1}=\dfrac{3z-19}{2}$ is equal to :

(1) 3 (2) 5

(3) 4 (4) 6

Ans. (1)

3. Let $\int\limits_0^x\sqrt{1-(y'(t))^2}\,dt=\int\limits_0^x y(t)dt$, $0\le x\le 3$, $y\ge 0$, $y(0)=0$. Then at $x=2$, $y''+y+1$ is equal to :

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) 1/2

Ans. (1)

4. Let z be a complex number such that the real part of $\dfrac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to :

(1) 12 (2) ∞

(3) 10 (4) 8

Ans. (1)

5. The area (in square units) of the region enclosed by the ellipse $x^2+3y^2=18$ in the first quadrant below the line $y=x$ is :

(1) $\sqrt{3}\pi+\dfrac{3}{4}$ (2) $\sqrt{3}\pi$

(3) $\sqrt{3}\pi-\dfrac{3}{4}$ (4) $\sqrt{3}\pi+1$

Ans. (2)

6. Let the foci of a hyperbola H coincide with the foci of the ellipse E : $\dfrac{(x-1)^2}{100}+\dfrac{(y-1)^2}{75}=1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2+2\beta^2$ is equal to :

(1) 242 (2) 225

(3) 237 (4) 205

Ans. (2)

7. Two vertices of a triangle ABC are A(3, –1) and B (–2, 3), and its orthocentre is P(1, 1). If the coordinates of the point C are (α, β) and the centre of the circle circumscribing the triangle PAB is (h, k), then the value of $(\alpha+\beta)+2(h+k)$ equals :

(1) 51 (2) 81

(3) 5 (4) 15

Ans. (3)

8. If the variance of the frequency distribution is 160, then the value of $c \in N$ is

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

(1) 5 (2) 8

(3) 7 (4) 6

Ans. (3)

9. Let the range of the function

$f(x) = \frac{1}{2+\sin 3x+\cos 3x}$, $x \in IR$ be [a, b].

If α and β are respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to :

(1) $\sqrt{2}$ (2) 2

(3) $\sqrt{\pi}$ (4) π

Ans. (1)

10. Between the following two statements :

Statement-I : Let $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}-\hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a}\times\vec{r} = \vec{a}\times\vec{b}$ and $\vec{a}.\vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement-II : In a triangle ABC, $\cos 2A + \cos 2B + \cos 2C \geq -\frac{3}{2}$.

(1) Both Statement-I and Statement-II are incorrect

(2) Statement-I is incorrect but Statement-II is correct

(3) Both Statement-I and Statement-II are correct

(4) Statement-I is correct but Statement-II is incorrect

Ans. (2)

11. $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{\int_{x^3}^{(\pi/2)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ is equal to :

(1) $\frac{9\pi^2}{8}$ (2) $\frac{11\pi^2}{10}$

(3) $\frac{3\pi^2}{2}$ (4) $\frac{5\pi^2}{9}$

Ans. (1)

12. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ is :

(1) 21/4 (2) 69/16

(3) 63/16 (4) 19/4

Ans. (1)

13. Let $B = \begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}$ and A be a 2×2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then $2\beta - \alpha$ is equal to :

(1) 16 (2) 2

(3) 8 (4) 10

Ans. (4)

14. If $\log_e y = 3\sin^{-1}x$, then $(1-x)^2 y'' - xy'$ at $x = \frac{1}{2}$ is equal to :

(1) $9e^{\pi/6}$ (2) $3e^{\pi/6}$

(3) $3e^{\pi/2}$ (4) $9e^{\pi/2}$

Ans. (4)

15. The integral $\int_{1/4}^{3/4}\cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$ is equal to:

(1) –1/2 (2) 1/4

(3) 1/2 (4) –1/4

Ans. (4)

16. Let a, ar, ar^2, ...be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3r^{3n} = 9747$, then a + 18r is equal to :

(1) 27 (2) 46

(3) 38 (4) 31

Ans. (4)

17. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, i = 2, 3, is equal to :

(1) 3/54

(2) 2/54

(3) 5/54

(4) 1/54

Ans. (3)

18. The value of the integral $\int_{-1}^{2} \log_e\left(x+\sqrt{x^2+1}\right)dx$ is :

(1) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(2) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{5}-\sqrt{2}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(4) $\sqrt{2}-\sqrt{5}+\log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

Ans. (2)

19. Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2-\sqrt{2}x-\sqrt{3}=0$. Let $P_n = \alpha^n-\beta^n$, $n \in N$. Then $\left(11\sqrt{3}-10\sqrt{2}\right) P_{10} + \left(11\sqrt{2}+10\right) P_{11} - 11P_{12}$ is equal to :

(1) $10\sqrt{2}P_9$

(2) $10\sqrt{3}P_9$

(3) $11\sqrt{2}P_9$

(4) $11\sqrt{3}P_9$

Ans. (2)

20. Let $\vec{a}=2\hat{i}+\alpha\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}+\hat{k}$, $\vec{c}=\beta\hat{j}-\hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the ordered pair (α, β) for which the area of the parallelogram of diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2).

Then $\alpha_1^2+\beta_1^2-\alpha_2\beta_2$ is equal to

(1) 17 (2) 24

(3) 21 (4) 19

Ans. (4)

SECTION-B

21. Consider the circle C : $x^2 + y^2 = 4$ and the parabola P : $y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to _____.

Ans. (80)

22. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then 16ab is equal to _____ .

Ans. (252)

23. For a differentiable function $f : IR \to IR$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in IR$, $f(0) = 1$ and $\lim\limits_{x\to-\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to_____.

Ans. (61)

24. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is _____.

Ans. (70)

25. Let A = {(x, y) : 2x + 3y = 23, x, y $\in$ N} and B = {x : (x, y) $\in$ A}. Then the number of one-one functions from A to B is equal to _____.

Ans. (24)

26. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L.

Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to _____.

Ans. (36)

27. The square of the distance of the image of the point (6, 1, 5) in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is _____.

Ans. (62)

28. If $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+1012}\right)$

$-\left(\frac{1}{2\cdot1} + \frac{1}{4\cdot3} + \frac{1}{6\cdot5} + ... + \frac{1}{2024\cdot2023}\right)$

$= \frac{1}{2024}$, then α is equal to-

Ans. (1011)

29. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$, is _____.

Ans. (0)

30. Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations AX = B has a negative solution (i.e., $x < 0$ and $y < 0$), be the interval (a, b).

Then $8\int_a^b |A|\,dm$ is equal to _____.

Ans. (450)

Are you targeting **JEE 2025 ?**

Ace it with **ALLEN**'s

Leader Course

Online Program 18 APRIL '24

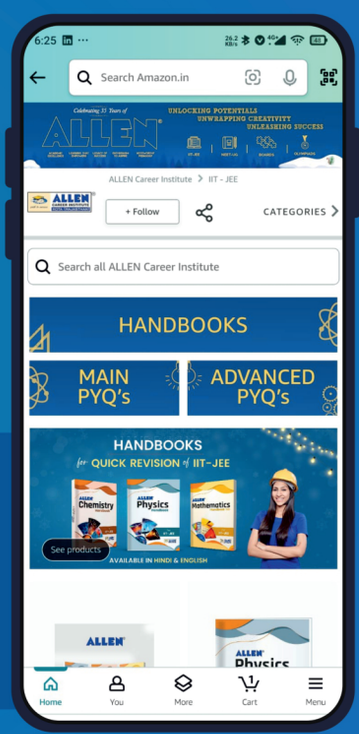
Offline Program 24 APRIL '24

ALLEN

Get The Latest

IIT-JEE Special Books

at Your Door Steps...!!



JOIN THE JOURNEY OF LEARNING

with

SCORE TEST PAPERS | HANDBOOKS

JEE-MAIN PYQ's | JEE-Adv. PYQ's

SHOP NOW

Available in
HINDI & ENGLISH