

FINAL JEE-MAIN EXAMINATION – APRIL, 2024

(Held On Monday 08th April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. The value of $k \in \mathbb{N}$ for which the integral

$$I_n = \int_0^1 (1-x^k)^n dx, n \in \mathbb{N}, \text{ satisfies } 147 I_{20} = 148 I_{21}$$

is :

- (1) 10 (2) 8
(3) 14 (4) 7

Ans. (4)

2. The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$ is :

- (1) $1 + \log_6(8)$ (2) $\log_8(6)$
(3) $1 + \log_8(6)$ (4) $\log_8(4)$

Ans. (3)

3. Let the circles $C_1 : (x - \alpha)^2 + (y - \beta)^2 = r_1^2$ and $C_2 : (x - 8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 and C_2 internally in the ratio 2 : 1, then $(\alpha + \beta) + 4(r_1^2 + r_2^2)$ equals

- (1) 110 (2) 130
(3) 125 (4) 145

Ans. (2)

4. Let $P(x, y, z)$ be a point in the first octant, whose projection in the xy -plane is the point Q . Let $OP = \gamma$; the angle between OQ and the positive x -axis be θ ; and the angle between OP and the positive z -axis be ϕ , where O is the origin. Then the distance of P from the x -axis is :

- (1) $\gamma\sqrt{1 - \sin^2 \phi \cos^2 \theta}$ (2) $\gamma\sqrt{1 + \cos^2 \theta \sin^2 \phi}$
(3) $\gamma\sqrt{1 - \sin^2 \theta \cos^2 \phi}$ (4) $\gamma\sqrt{1 + \cos^2 \phi \sin^2 \theta}$

Ans. (1)

5. The number of critical points of the function $f(x) = (x - 2)^{2/3} (2x + 1)$ is :

- (1) 2 (2) 0
(3) 1 (4) 3

Ans. (1)

6. Let $f(x)$ be a positive function such that the area bounded by $y = f(x)$, $y = 0$ from $x = 0$ to $x = a > 0$ is $e^{-a} + 4a^2 + a - 1$. Then the differential equation, whose general solution is $y = c_1 f(x) + c_2$, where c_1 and c_2 are arbitrary constants, is :

- (1) $(8e^x - 1) \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
(2) $(8e^x + 1) \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
(3) $(8e^x + 1) \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
(4) $(8e^x - 1) \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Ans. (3)

7. Let $f(x) = 4\cos^3 x + 3\sqrt{3} \cos^2 x - 10$. The number of points of local maxima of f in interval $(0, 2\pi)$ is:

- (1) 1 (2) 2
(3) 3 (4) 4

Ans. (2)

8. Let $A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 5 & b \end{bmatrix}$. If $A^3 = 4A^2 - A - 21I$, where

I is the identity matrix of order 3×3 , then $2a + 3b$ is equal to :

- (1) -10 (2) -13
(3) -9 (4) -12

Ans. (2)



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9. If the shortest distance between the lines
 $L_1 : \vec{r} = (2 + \lambda)\hat{i} + (1 - 3\lambda)\hat{j} + (3 + 4\lambda)\hat{k}, \lambda \in \mathbb{R}$
 $L_2 : \vec{r} = 2(1 + \mu)\hat{i} + 3(1 + \mu)\hat{j} + (5 + \mu)\hat{k}, \mu \in \mathbb{R}$
 is $\frac{m}{\sqrt{n}}$, where $\gcd(m, n) = 1$, then the value of
 $m + n$ equals.

- (1) 384 (2) 387
 (3) 377 (4) 390

Ans. (2)

10. Let the sum of two positive integers be 24. If the probability, that their product is not less than $\frac{3}{4}$ times their greatest positive product, is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ equals :

- (1) 9 (2) 11
 (3) 8 (4) 10

Ans. (4)

11. If $\sin x = -\frac{3}{5}$, where $\pi < x < \frac{3\pi}{2}$, then $80(\tan^2 x - \cos x)$ is equal to :

- (1) 109 (2) 108
 (3) 18 (4) 19

Ans. (1)

12. Let $I(x) = \int \frac{6}{\sin^2 x (1 - \cot x)^2} dx$. If $I(0) = 3$, then

$I\left(\frac{\pi}{12}\right)$ is equal to :

- (1) $\sqrt{3}$ (2) $3\sqrt{3}$
 (3) $6\sqrt{3}$ (4) $2\sqrt{3}$

Ans. (2)

13. The equations of two sides AB and AC of a triangle ABC are $4x + y = 14$ and $3x - 2y = 5$, respectively. The point $\left(2, -\frac{4}{3}\right)$ divides the third side BC internally in the ratio 2 : 1. The equation of the side BC is :

- (1) $x - 6y - 10 = 0$ (2) $x - 3y - 6 = 0$
 (3) $x + 3y + 2 = 0$ (4) $x + 6y + 6 = 0$

Ans. (3)

14. Let $[t]$ be the greatest integer less than or equal to t . Let A be the set of all prime factors of 2310 and

$f : A \rightarrow \mathbb{Z}$ be the function $f(x) = \left\lfloor \log_2 \left(x^2 + \left\lceil \frac{x^3}{5} \right\rceil \right) \right\rfloor$.

The number of one-to-one functions from A to the range of f is :

- (1) 20 (2) 120
 (3) 25 (4) 24

Ans. (2)

15. Let z be a complex number such that $|z + 2| = 1$ and $\operatorname{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$. Then the value of $\left| \operatorname{Re}\left(\overline{z+2}\right) \right|$ is :

- (1) $\frac{\sqrt{6}}{5}$ (2) $\frac{1 + \sqrt{6}}{5}$
 (3) $\frac{24}{5}$ (4) $\frac{2\sqrt{6}}{5}$

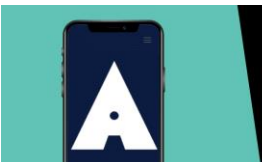
Ans. (4)

16. If the set $R = \{(a, b) ; a + 5b = 42, a, b \in \mathbb{N}\}$ has m elements and $\sum_{n=1}^m (1 + i^{n!}) = x + iy$, where

$I = \sqrt{-1}$, then the value of $m + x + y$ is :

- (1) 8 (2) 12
 (3) 4 (4) 5

Ans. (2)



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17. For the function $f(x) = (\cos x) - x + 1$, $x \in \mathbb{R}$, between the following two statements

(S1) $f(x) = 0$ for only one value of x is $[0, \pi]$.

(S2) $f(x)$ is decreasing in $\left[0, \frac{\pi}{2}\right]$ and increasing in $\left[\frac{\pi}{2}, \pi\right]$.

- (1) Both (S1) and (S2) are correct
- (2) Only (S1) is correct
- (3) Both (S1) and (S2) are incorrect
- (4) Only (S2) is correct

Ans. (2)

18. The set of all α , for which the vector $\vec{a} = \alpha\hat{i} + 6\hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 2\alpha\hat{k}$ are inclined at an obtuse angle for all $t \in \mathbb{R}$ is :

- (1) $[0, 1)$
- (2) $(-2, 0]$
- (3) $\left(-\frac{4}{3}, 0\right]$
- (4) $\left(-\frac{4}{3}, 1\right)$

Ans. (3)

19. Let $y = y(x)$ be the solution of the differential equation $(1 + y^2)e^{\tan x} dx + \cos^2 x(1 + e^{2\tan x}) dy = 0$, $y(0) = 1$. Then $y\left(\frac{\pi}{4}\right)$ is equal to :

- (1) $\frac{2}{e}$
- (2) $\frac{1}{e^2}$
- (3) $\frac{1}{e}$
- (4) $\frac{2}{e^2}$

Ans. (3)

20. Let $H : \frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the hyperbola, whose eccentricity is $\sqrt{3}$ and the length of the latus rectum is $4\sqrt{3}$. Suppose the point $(\alpha, 6)$, $\alpha > 0$ lies on H . If β is the product of the focal distances of the point $(\alpha, 6)$, then $\alpha^2 + \beta$ is equal to :

- (1) 170
- (2) 171
- (3) 169
- (4) 172

Ans. (2)

SECTION-B

21. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$. If the sum of the diagonal elements of A^{13} is 3^n , then n is equal to _____.

Ans. (7)

22. If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumcentre and orthocentre respectively are $(3, 4)$ and $(-6, -8)$, then the value of $|a - b|$ is _____.

Ans. (16)

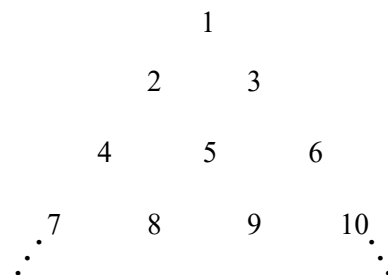
23. Three balls are drawn at random from a bag containing 5 blue and 4 yellow balls. Let the random variables X and Y respectively denote the number of blue and Yellow balls. If \bar{X} and \bar{Y} are the means of X and Y respectively, then $7\bar{X} + 4\bar{Y}$ is equal to _____.

Ans. (17)

24. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to _____.

Ans. (36)

25. Let the positive integers be written in the form :



If the k^{th} row contains exactly k numbers for every natural number k , then the row in which the number 5310 will be, is _____.

Ans. (103)

26. If the range of $f(\theta) = \frac{\sin^4 \theta + 3 \cos^2 \theta}{\sin^4 \theta + \cos^2 \theta}$, $\theta \in \mathbb{R}$ is $[\alpha, \beta]$, then the sum of the infinite G.P., whose first term is 64 and the common ratio is $\frac{\alpha}{\beta}$, is equal to _____.

Ans. (96)

27. Let $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1) {}^n C_r$
 and $\beta = \left(\sum_{r=0}^n \frac{{}^n C_r}{r+1} \right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$,
 then the value of n is _____.

Ans. (5)

28. Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$, $\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If \vec{r} is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{|593\vec{r} + 67\vec{a}|^2}{(593)^2}$ is equal to _____.

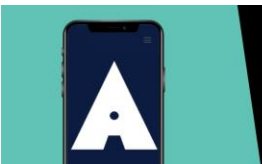
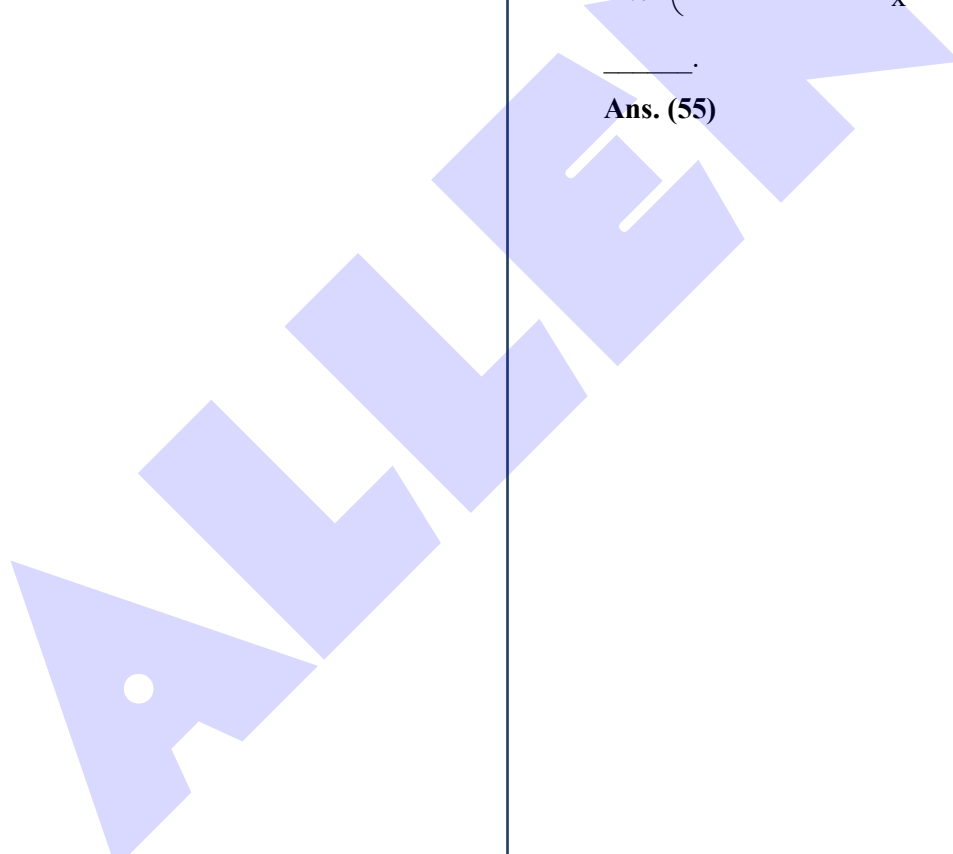
Ans. (569)

29. Let the area of the region enclosed by the curve $y = \min\{\sin x, \cos x\}$ and the x -axis between $x = -\pi$ to $x = \pi$ be A . Then A^2 is equal to _____.

Ans. (16)

30. The value of $\lim_{x \rightarrow 0} 2 \left(\frac{1 - \cos x \sqrt{\cos 2x} \sqrt[3]{\cos 3x} \dots \sqrt[10]{\cos 10x}}{x^2} \right)$ is _____.

Ans. (55)



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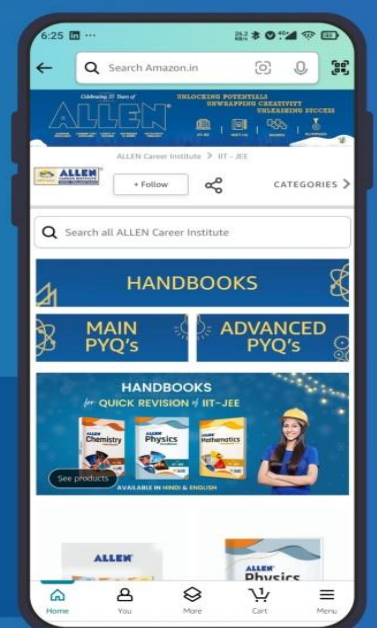
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