FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Monday 08th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. If the image of the point (-4, 5) in the line $x + 2y = 2$ lies on the circle $(x + 4)^2 + (y-3)^2 = r^2$, then r is equal lo :

(1) 1 (2) 2

(3) 75 (4) 3

Ans. (2)

2. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let $\vec{r}$ be a unit vector along $\vec{b} + \vec{c}$. If $\vec{r} \cdot \vec{a} = 3$, then 3λ is equal to :

(1) 27 (2) 25

(3) 25 (4) 21

Ans. (2)

3. If $\alpha \neq a$, $\beta \neq b$, $\gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$, then $\frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c}$ is equal to :

(1) 2 (2) 3

(3) 0 (4) 1

Ans. (3)

4. In an increasing geometric progression ol positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is 49. Then the sum of the 4th, 6th and 8th terms is :-

(1) 96 (2) 78

(3) 91 (4) 84

Ans. (3)

5. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to :

(1) 175 (2) 181

(3) 177 (4) 179

Ans. (4)

6. The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is purely imaginary, is equal to

(1) 2π (2) 3π

(3) 5π (4) 4π

Ans. (2)

7. If the system of equations $x + 4y - z = \lambda$, $7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu. + 3\lambda)$ is equal to :

(1) 2 (2) –3

(3) 3 (4) –2

Ans. (2)

8. If the shortest distance between the lines $\frac{x - \lambda}{2} = \frac{y - 4}{3} = \frac{z - 3}{4}$ and $\frac{x - 2}{4} = \frac{y - 4}{6} = \frac{z - 7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of λ is :

(1) $-\frac{13}{25}$ (2) $\frac{13}{25}$

(3) 1 (4) –1

Ans. (3)

9. If the value of $\frac{3\cos 36^\circ + 5\sin 18^\circ}{5\cos 36^\circ - 3\sin 18^\circ}$ is $\frac{a\sqrt{5} - b}{c}$, where a, b, c are natural numbers and gcd(a, c) = 1, then a + b + c is equal to :

(1) 50 (2) 40

(3) 52 (4) 54

Ans. (3)

10. Let y = y(x) be the solution curve of the differential equation $\sec y \frac{dy}{dx} + 2x\sin y = x^3\cos y$, y(1) = 0. Then $y\left(\sqrt{3}\right)$ is equal to :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{12}$

Ans. (3)

11. The area of the region in the first quadrant inside the circle $x^2 + y^2 = 8$ and outside the pnrabola $y^2 = 2x$ is equal to :

(1) $\frac{\pi}{2} - \frac{1}{3}$ (2) $\pi - \frac{2}{3}$

(3) $\frac{\pi}{2} - \frac{2}{3}$ (4) $\pi - \frac{1}{3}$

Ans. (2)

12. If the line segment joining the points (5, 2) and (2, a) subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is :

(1) 6 (2) 8

(3) 2 (4) 4

Ans. (4)

13. Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that

$(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b})$.

If $(2\vec{a} + 3\vec{b}).\vec{c} = 1670$, then $|\vec{c}|^2$ is equal to :

(1) 1627 (2) 1618

(3) 1600 (4) 1609

Ans. (2)

14. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, a > 0 has a local maximum at x = α and a local minimum $x = \alpha^2$, then α and α^2 are the roots of the equation :

(1) $x^2 - 6x + 8 = 0$ (2) $8x^2 + 6x - 8 = 0$

(3) $8x^2 - 6x + 1 = 0$ (4) $x^2 + 6x + 8 = 0$

Ans. (1)

15. There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is :

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{5}{12}$

Ans. (1)

16. Let $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. Then e^{α} and $e^{-\alpha}$ are the roots of the equation :

(1) $2x^2 - 5x + 2 = 0$ (2) $x^2 - 2x - 8 = 0$

(3) $2x^2 - 5x - 2 = 0$ (4) $x^2 + 2x - 8 = 0$

Ans. (1)

17. Let $f(x) = \begin{cases} -a & \text{if } -a \le x \le 0 \\ x+a & \text{if } 0 < x \le a \end{cases}$

where $a > 0$ and $g(x) = (f|x|) - |f(x)|)/2$.

Then the function $g : [-a, a] \to [-a, a]$ is

(1) neither one-one nor onto.

(2) both one-one and onto.

(3) one-one.

(4) onto

Ans. (1)

18. Let A= {2, 3, 6, 8, 9, 11} and B = {1, 4, 5, 10, 15} Let R be a relation on A × B define by (a, b)R(c, d) if and only if 3ad – 7bc is an even integer. Then the relation R is

(1) reflexive but not symmetric.

(2) transitive but not symmetric.

(3) reflexive and symmetric but not transitive.

(4) an equivalence relation.

Ans. (3)

19. For $a, b > 0$, let

$$f(x) = \begin{cases} \dfrac{\tan((a+1)x) + b\tan x}{x}, & x < 0 \\ 3, & x = 0 \\ \dfrac{\sqrt{ax + b^2x^2} - \sqrt{ax}}{b\sqrt{a}\, x\sqrt{x}}, & x > 0 \end{cases}$$

be a continous function at $x = 0$. Then $\frac{b}{a}$ is equal to

(1) 5 (2) 4

(3) 8 (4) 6

Ans. (4)

20. If the term independent of x in the expansion of $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$ is 105, then a^2 is equal to :

(1) 4 (2) 9

(3) 6 (4) 2

Ans. (1)

SECTION-B

21. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is _____ .

Ans. (128)

22. If $\alpha = \lim\limits_{x \to 0^+} \left(\dfrac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}} \right)$ and $\beta = \lim\limits_{x \to 0} (1 + \sin x)^{\frac{1}{2}\cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then $12 \log_e(a + b)$ is equal to _____ .

Ans. (6)

23. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x-axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S. if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to -

Ans. (40)

24. Let $P(\alpha, \beta, \gamma)$ be the image of the point Q(l, 6, 4) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to _____ .

Ans. (11)

25. An arithmetic progression is written in the following way

2

5 8

11 14 17

20 23 26 29

_____

_____

The sum of all the terms of the 10^{th} row is _____ .

Ans. (1505)

26. The number of distinct real roots of the equation $|x + 1|\,|x + 3| - 4\,|x + 2| + 5 = 0$, is _____ .

Ans. (2)

27. Let a ray of light passing through the point (3, 10) reflects on the line $2x + y = 6$ and the reflected ray passes through the point (7, 2). If the equation of the incident ray is $ax + by + 1 = 0$, then $a^2 + b^2 + 3ab$ is equal to_.

Ans. (1)

28. Let a, b, c $\in$ N and $a < b < c$. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\frac{136}{5}$, respectively. Then $2a + b - c$ is equal to _____ .

Ans. (33)

29. Lei $\alpha|x| = |y|e^{xy-\beta}$, $\alpha, \beta \in N$ be the solution of the differential equation $xdy - ydx + xy(xdy+ydx) = 0$, $y(l) = 2$. Then $\alpha + \beta$ is equal to _

Ans. (4)

30. If $\int \frac{1}{\sqrt[5]{(x-1)^4\,(x+3)^6}}dx = A\left(\frac{\alpha x - 1}{\beta x + 3}\right)^B + C$, where C is the constant of integration, then the value of $\alpha + \beta + 20AB$ is _____ .

Ans. (7)