

FINAL JEE-MAIN EXAMINATION – APRIL, 2024
(Held On Monday 08th April, 2024)
TIME : 3 : 00 PM to 6 : 00 PM
MATHEMATICS
TEST PAPER WITH ANSWER
SECTION-A

1. If the image of the point $(-4, 5)$ in the line $x + 2y = 2$ lies on the circle $(x + 4)^2 + (y - 3)^2 = r^2$, then r is equal to :

- (1) 1 (2) 2
(3) 75 (4) 3

Ans. (2)

2. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ be three vectors. Let \vec{r} be a unit vector along $\vec{b} + \vec{c}$. If $\vec{r} \cdot \vec{a} = 3$, then 3λ is equal to :

- (1) 27 (2) 25
(3) 25 (4) 21

Ans. (2)

3. If $\alpha \neq a$, $\beta \neq b$, $\gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$, then

$\frac{a}{\alpha - a} + \frac{b}{\beta - b} + \frac{\gamma}{\gamma - c}$ is equal to :

- (1) 2 (2) 3
(3) 0 (4) 1

Ans. (3)

4. In an increasing geometric progression of positive terms, the sum of the second and sixth terms is $\frac{70}{3}$ and the product of the third and fifth terms is

49. Then the sum of the 4th, 6th and 8th terms is :-

- (1) 96 (2) 78
(3) 91 (4) 84

Ans. (3)

5. The number of ways five alphabets can be chosen from the alphabets of the word MATHEMATICS, where the chosen alphabets are not necessarily distinct, is equal to :

- (1) 175 (2) 181
(3) 177 (4) 179

Ans. (4)

6. The sum of all possible values of $\theta \in [-\pi, 2\pi]$, for which $\frac{1 + i \cos \theta}{1 - 2i \cos \theta}$ is purely imaginary, is equal to

- (1) 2π (2) 3π
(3) 5π (4) 4π

Ans. (2)

7. If the system of equations $x + 4y - z = \lambda$, $7x + 9y + \mu z = -3$, $5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to :

- (1) 2 (2) -3
(3) 3 (4) -2

Ans. (2)

8. If the shortest distance between the lines $\frac{x - \lambda}{2} = \frac{y - 4}{3} = \frac{z - 3}{4}$ and $\frac{x - 2}{4} = \frac{y - 4}{6} = \frac{z - 7}{8}$ is $\frac{13}{\sqrt{29}}$, then a value of

λ is :

- (1) $-\frac{13}{25}$ (2) $\frac{13}{25}$
(3) 1 (4) -1

Ans. (3)


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9. If the value of $\frac{3 \cos 36^\circ + 5 \sin 18^\circ}{5 \cos 36^\circ - 3 \sin 18^\circ}$ is $\frac{a\sqrt{5} - b}{c}$,

where a, b, c are natural numbers and $\gcd(a, c) = 1$,

then $a + b + c$ is equal to :

- (1) 50 (2) 40
 (3) 52 (4) 54

Ans. (3)

10. Let $y = y(x)$ be the solution curve of the differential equation $\sec y \frac{dy}{dx} + 2x \sin y = x^3 \cos y$,

$y(1) = 0$. Then $y(\sqrt{3})$ is equal to :

- (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$
 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{12}$

Ans. (3)

11. The area of the region in the first quadrant inside the circle $x^2 + y^2 = 8$ and outside the parabola $y^2 = 2x$ is equal to :

- (1) $\frac{\pi}{2} - \frac{1}{3}$ (2) $\pi - \frac{2}{3}$
 (3) $\frac{\pi}{2} - \frac{2}{3}$ (4) $\pi - \frac{1}{3}$

Ans. (2)

12. If the line segment joining the points $(5, 2)$ and $(2, a)$ subtends an angle $\frac{\pi}{4}$ at the origin, then the absolute value of the product of all possible values of a is :

- (1) 6 (2) 8
 (3) 2 (4) 4

Ans. (4)

13. Let $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 11\hat{i} - \hat{j} + \hat{k}$ and \vec{c} be a vector such that

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{c} \times (-2\vec{a} + 3\vec{b}).$$

If $(2\vec{a} + 3\vec{b}) \cdot \vec{c} = 1670$, then $|\vec{c}|^2$ is equal to :

- (1) 1627 (2) 1618
 (3) 1600 (4) 1609

Ans. (2)

14. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, $a > 0$ has a local maximum at $x = \alpha$ and a local minimum $x = \alpha^2$, then α and α^2 are the roots of the equation :

- (1) $x^2 - 6x + 8 = 0$ (2) $8x^2 + 6x - 8 = 0$
 (3) $8x^2 - 6x + 1 = 0$ (4) $x^2 + 6x + 8 = 0$

Ans. (1)

15. There are three bags X, Y and Z. Bag X contains 5 one-rupee coins and 4 five-rupee coins; Bag Y contains 4 one-rupee coins and 5 five-rupee coins and Bag Z contains 3 one-rupee coins and 6 five-rupee coins. A bag is selected at random and a coin drawn from it at random is found to be a one-rupee coin. Then the probability, that it came from bag Y, is :

- (1) $\frac{1}{3}$ (2) $\frac{1}{2}$
 (3) $\frac{1}{4}$ (4) $\frac{5}{12}$

Ans. (1)

16. Let $\int_{\alpha}^{\log_e 4} \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$. Then e^α and $e^{-\alpha}$ are the

roots of the equation :

- (1) $2x^2 - 5x + 2 = 0$ (2) $x^2 - 2x - 8 = 0$
 (3) $2x^2 - 5x - 2 = 0$ (4) $x^2 + 2x - 8 = 0$

Ans. (1)



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17. Let $f(x) = \begin{cases} -a & \text{if } -a \leq x \leq 0 \\ x+a & \text{if } 0 < x \leq a \end{cases}$
 where $a > 0$ and $g(x) = (f(|x|) - |f(x)|)/2$.
 Then the function $g : [-a, a] \rightarrow [-a, a]$ is

- (1) neither one-one nor onto.
- (2) both one-one and onto.
- (3) one-one.
- (4) onto

Ans. (1)

18. Let $A = \{2, 3, 6, 8, 9, 11\}$ and $B = \{1, 4, 5, 10, 15\}$
 Let R be a relation on $A \times B$ define by $(a, b)R(c, d)$
 if and only if $3ad - 7bc$ is an even integer. Then
 the relation R is

- (1) reflexive but not symmetric.
- (2) transitive but not symmetric.
- (3) reflexive and symmetric but not transitive.
- (4) an equivalence relation.

Ans. (3)

19. For $a, b > 0$, let

$$f(x) = \begin{cases} \frac{\tan((a+1)x) + b \tan x}{3}, & x < 0 \\ \frac{x}{\sqrt{ax + b^2x^2} - \sqrt{ax}}, & x = 0 \\ \frac{b\sqrt{ax}\sqrt{x}}{b\sqrt{ax}\sqrt{x}}, & x > 0 \end{cases}$$

be a continuous function at $x = 0$. Then $\frac{b}{a}$ is equal

to

- (1) 5
- (2) 4
- (3) 8
- (4) 6

Ans. (4)

20. If the term independent of x in the expansion of

$$\left(\sqrt{ax^2} + \frac{1}{2x^3}\right)^{10}$$

is 105, then a^2 is equal to :

- (1) 4
- (2) 9
- (3) 6
- (4) 2

Ans. (1)

SECTION-B

21. Let A be the region enclosed by the parabola $y^2 = 2x$ and the line $x = 24$. Then the maximum area of the rectangle inscribed in the region A is _____.

Ans. (128)

22. If $\alpha = \lim_{x \rightarrow 0^+} \left(\frac{e^{\sqrt{\tan x}} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}} \right)$ and

$\beta = \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{2} \cot x}$ are the roots of the quadratic equation $ax^2 + bx - \sqrt{e} = 0$, then $12 \log_e(a + b)$ is equal to _____.

Ans. (6)

23. Let S be the focus of the hyperbola $\frac{x^2}{3} - \frac{y^2}{5} = 1$, on the positive x -axis. Let C be the circle with its centre at $A(\sqrt{6}, \sqrt{5})$ and passing through the point S . if O is the origin and SAB is a diameter of C then the square of the area of the triangle OSB is equal to -

Ans. (40)

24. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then $2\alpha + \beta + \gamma$ is equal to _____.


Ans. (11)

25. An arithmetic progression is written in the following way

		2		
		5	8	
	11	14	17	
20	23	26	29	

The sum of all the terms of the 10th row is _____.

Ans. (1505)



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26. The number of distinct real roots of the equation $|x + 1| |x + 3| - 4|x + 2| + 5 = 0$, is _____ .

Ans. (2)

27. Let a ray of light passing through the point (3, 10) reflects on the line $2x + y = 6$ and the reflected ray passes through the point (7, 2). If the equation of the incident ray is $ax + by + 1 = 0$, then $a^2 + b^2 + 3ab$ is equal to _.

Ans. (1)

28. Let $a, b, c \in \mathbb{N}$ and $a < b < c$. Let the mean, the mean deviation about the mean and the variance of the 5 observations 9, 25, a, b, c be 18, 4 and $\frac{136}{5}$, respectively. Then $2a + b - c$ is equal to _____ .

Ans. (33)

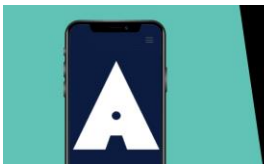
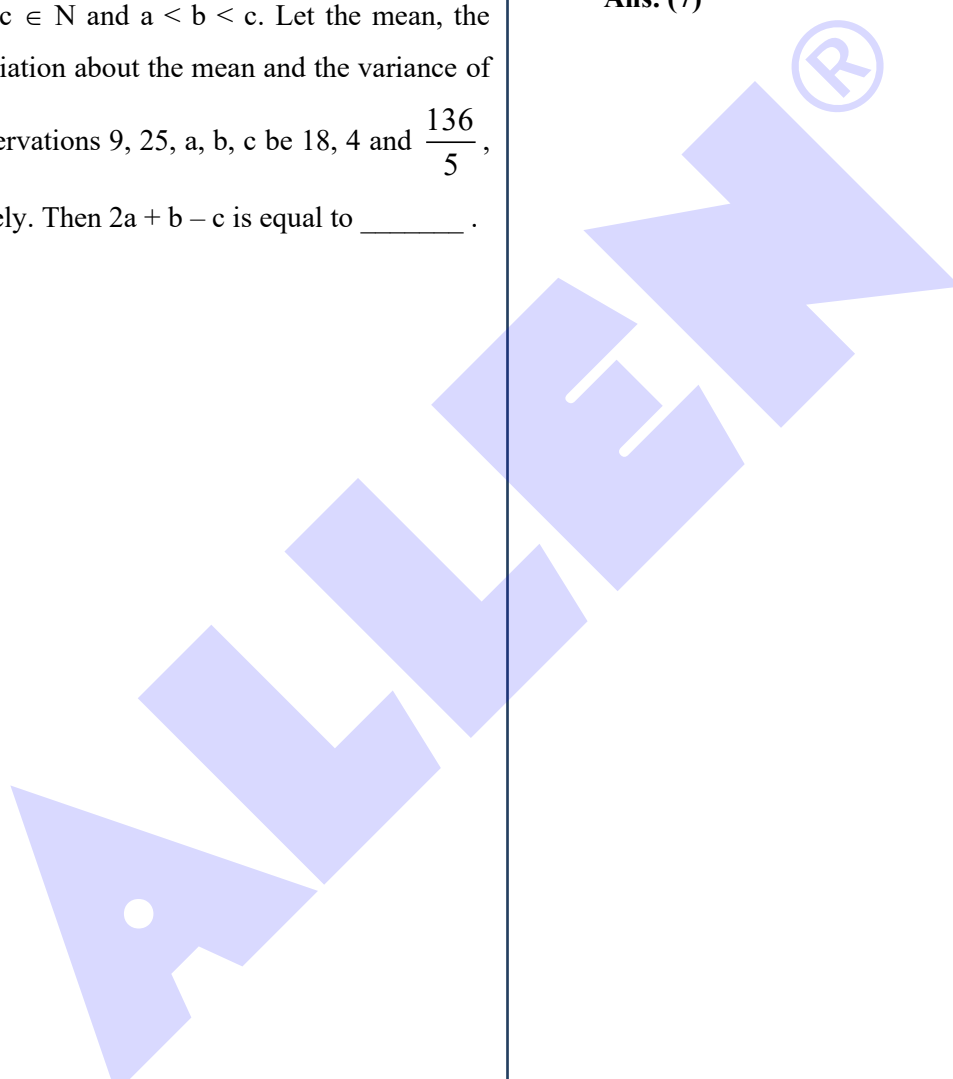
29. Let $\alpha|x| = |y|e^{xy-\beta}$, $\alpha, \beta \in \mathbb{N}$ be the solution of the differential equation $xydy - ydx + xy(xdy+ydx) = 0$, $y(1) = 2$. Then $\alpha + \beta$ is equal to _

Ans. (4)

30. If $\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}} dx = A \left(\frac{\alpha x - 1}{\beta x + 3} \right)^B + C$,

where C is the constant of integration, then the value of $\alpha + \beta + 20AB$ is _____ .

Ans. (7)



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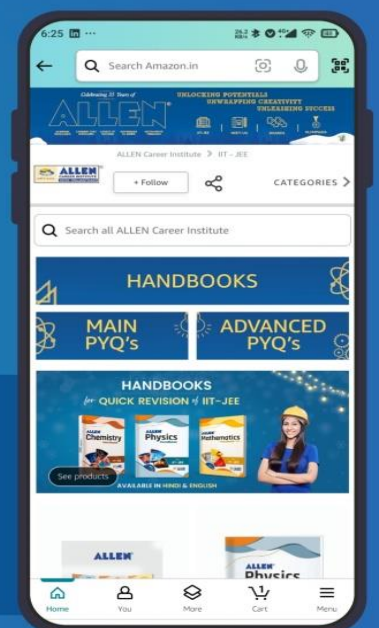
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