

FINAL JEE-MAIN EXAMINATION – APRIL, 2024
(Held On Saturday 06th April, 2024)
TIME : 9 : 00 AM to 12 : 00 NOON
MATHEMATICS
TEST PAPER WITH ANSWER
SECTION-A

1. If $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

(1) $f''(0) = 1$ (2) $f''\left(\frac{2}{\pi}\right) = \frac{24 - \pi^2}{2\pi}$

(3) $f''\left(\frac{2}{\pi}\right) = \frac{12 - \pi^2}{2\pi}$ (4) $f''(0) = 0$

Ans. (2)

2. If $A(3,1,-1)$, $B\left(\frac{5}{3}, \frac{7}{3}, \frac{1}{3}\right)$, $C(2,2,1)$ and

$D\left(\frac{10}{3}, \frac{2}{3}, \frac{-1}{3}\right)$ are the vertices of a quadrilateral

ABCD, then its area is

(1) $\frac{4\sqrt{2}}{3}$ (2) $\frac{5\sqrt{2}}{3}$

(3) $2\sqrt{2}$ (4) $\frac{2\sqrt{2}}{3}$

Ans. (1)

3. $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2} dx$ is equal to

(1) $1/12$ (2) $1/9$

(3) $1/6$ (4) $1/3$

Ans. (3)

4. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On respectively, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(1) $\sqrt{3.86}$ (2) 1.8

(3) $\sqrt{3.96}$ (4) 1.94

Ans. (3)

5. The function $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}$, $x \in \mathbb{R}$ is

(1) both one-one and onto.

(2) onto but not one-one.

(3) neither one-one nor onto.

(4) one-one but not onto.

NTA Ans. (3)
Ans. Bonus

6. Let $A = \{n \in [100, 700] \cap \mathbb{N} : n \text{ is neither a multiple of 3 nor a multiple of 4}\}$. Then the number of elements in A is

(1) 300 (2) 280

(3) 310 (4) 290

Ans. (1)

7. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C?

(1) (2, 4) (2) (1, 2)

(3) (2, 2) (4) (1, 1)

Ans. (1)

8. For $\alpha, \beta \in \mathbb{R}$ and a natural number n, let

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

Then $2A_{10} - A_8$ is

(1) $4\alpha + 2\beta$ (2) $2\alpha + 4\beta$

(3) $2n$ (4) 0

Ans. (1)


Download the new **ALLEN app**
 & enroll for **Online Programs**

CLICK HERE TO
 DOWNLOAD

9. The shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is

- (1) $6\sqrt{3}$ (2) $4\sqrt{3}$
 (3) $5\sqrt{3}$ (4) $8\sqrt{3}$

Ans. (2)

10. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then $126p$ is

- (1) 54 (2) 64
 (3) 66 (4) 56

Ans. (1)

11. Let, α, β be the distinct roots of the equation $x^2 - (t^2 - 5t + 6)x + 1 = 0, t \in \mathbb{R}$ and $a_n = \alpha^n + \beta^n$.

Then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is

- (1) $1/4$ (2) $-1/2$
 (3) $-1/4$ (4) $1/2$

Ans. (3)

12. Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \dots, 20\}$ be given by $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2 , respectively, in order to make the relations symmetric, then $M + N$ equals

- (1) 8 (2) 16
 (3) 12 (4) 10

Ans. (4)

13. Let a variable line of slope $m > 0$ passing through the point $(4, -9)$ intersect the coordinate axes at the points A and B. the minimum value of the sum of the distances of A and B from the origin is

- (1) 25 (2) 30
 (3) 15 (4) 10

Ans. (1)

14. The interval in which the function $f(x) = x^x, x > 0$, is strictly increasing is

- (1) $\left(0, \frac{1}{e}\right]$ (2) $\left[\frac{1}{e^2}, 1\right)$
 (3) $(0, \infty)$ (4) $\left[\frac{1}{e}, \infty\right)$

Ans. (4)

15. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then $m + n^2$ is equal to

- (1) 396 (2) 408
 (3) 312 (4) 414

Ans. (2)

16. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

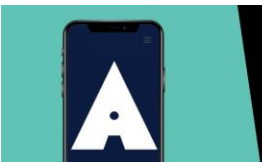
- (1) 24 (2) 56
 (3) 16 (4) 48

Ans. (3)

17. Let $y = y(x)$ be the solution of the differential equation $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}, y(1) = 0$. Then $y(0)$ is

- (1) $\frac{1}{4}(e^{\pi/2} - 1)$ (2) $\frac{1}{2}(1 - e^{\pi/2})$
 (3) $\frac{1}{4}(1 - e^{\pi/2})$ (4) $\frac{1}{2}(e^{\pi/2} - 1)$

Ans. (2)



Download the new **ALLEN** app
 & enroll for **Online Programs**

CLICK HERE TO
 DOWNLOAD

18. Let $y = y(x)$ be the solution of the differential equation $(2x \log_e x) \frac{dy}{dx} + 2y = \frac{3}{x} \log_e x$, $x > 0$ and $y(e^{-1}) = 0$. Then, $y(e)$ is equal to
- (1) $-\frac{3}{2e}$ (2) $-\frac{2}{3e}$
 (3) $-\frac{3}{e}$ (4) $-\frac{2}{e}$

Ans. (3)

19. Let the area of the region enclosed by the curves $y = 3x$, $2y = 27 - 3x$ and $y = 3x - x\sqrt{x}$ be A. Then $10A$ is equal to
- (1) 184 (2) 154
 (3) 172 (4) 162

Ans. (4)

20. Let $f: (-\infty, \infty) - \{0\} \rightarrow \mathbb{R}$ be a differentiable function such that $f'(1) = \lim_{a \rightarrow \infty} a^2 f\left(\frac{1}{a}\right)$.

Then $\lim_{a \rightarrow \infty} \frac{a(a+1)}{2} \tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2 \log_e a$ is equal to

- (1) $\frac{3}{2} + \frac{\pi}{4}$ (2) $\frac{3}{8} + \frac{\pi}{4}$
 (3) $\frac{5}{2} + \frac{\pi}{8}$ (4) $\frac{3}{4} + \frac{\pi}{8}$

Ans. (3)

SECTION-B

21. Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in \mathbb{R}$. If $x(\alpha, 1, 2) + y(1, \beta, 2) + z(2, 3, \gamma) = (0, 0, 0)$ for some $x, y, z \in \mathbb{R}$, $xyz \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to _____

Ans. (55)

22. Let a conic C pass through the point $(4, -2)$ and $P(x, y)$, $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(3, -5)$. If the focal distance of the point $(7, 1)$ on C is d, then $12d$ equals _____.

Ans. (75)

23. Let $r_k = \frac{\int_0^1 (1-x^7)^k dx}{\int_0^1 (1-x^7)^{k+1} dx}$, $k \in \mathbb{N}$. Then the value of $\sum_{k=1}^{10} \frac{1}{7(r_k - 1)}$ is equal to _____.

Ans. (65)

24. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and

$$(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16} m.$$

Then the value of m is _____.

Ans. (221)

25. Let L_1, L_2 be the lines passing through the point $P(0, 1)$ and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the points on the lines L_1 and L_2 such that the ΔPQR is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2 , then $16(m_1^2 + m_2^2)$ is equal to _____.

Ans. (68)

26. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6(n^3 + x^2 + y)$ is equal to _____.

Ans. (806)



Download the new ALLEN app & enroll for Online Programs

CLICK HERE TO DOWNLOAD

27. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3 T_{r-1} + 6^r$, $r = 2, 3, \dots, n$. If the sum of the first n terms of this series is $\frac{1}{5}(n^2 - 12n + 39)$ $(4 \cdot 6^n - 5 \cdot 3^n + 1)$. Then n is equal to _____.

Ans. (6)

28. For $n \in \mathbb{N}$, if $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{-1}n = \frac{\pi}{4}$, then n is equal to _____.

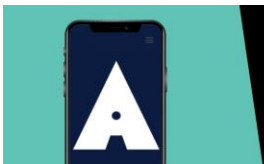
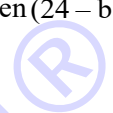
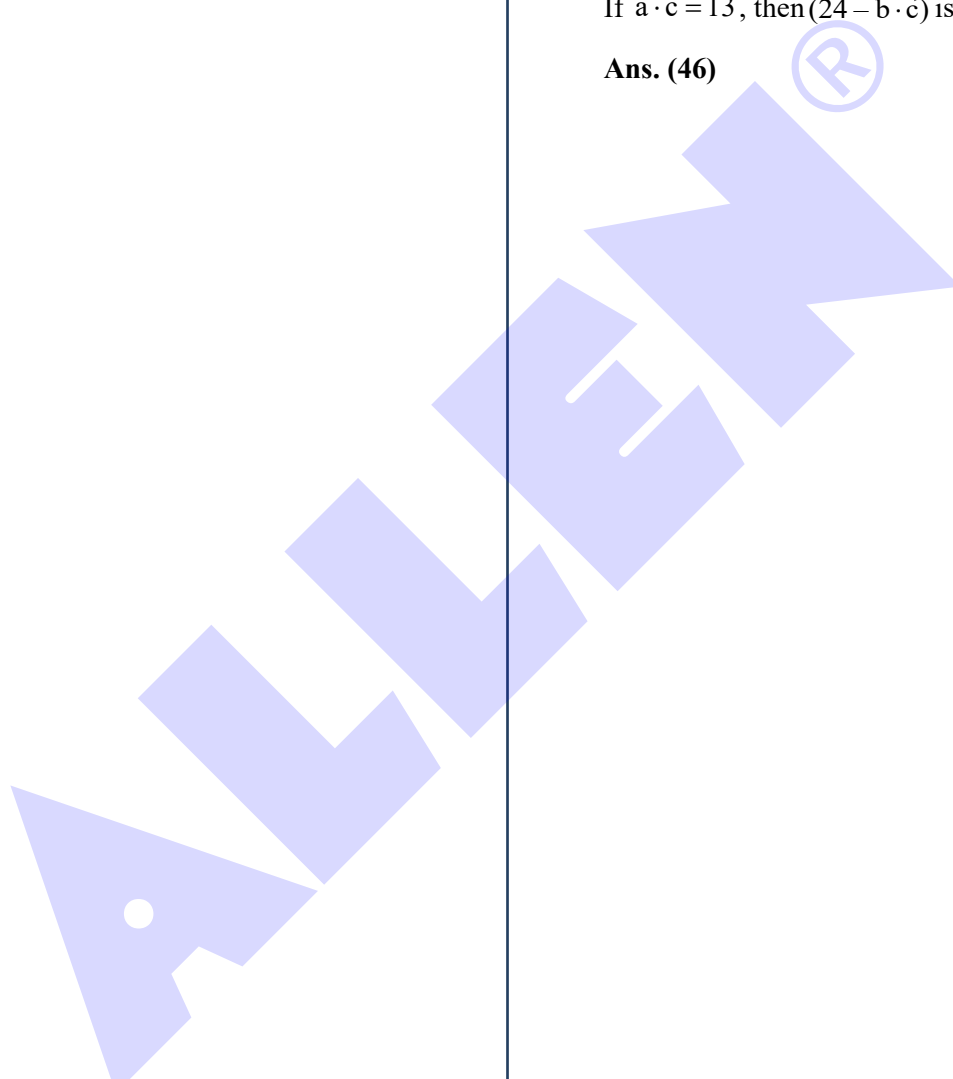
Ans. (47)

29. Let P be the point $(10, -2, -1)$ and Q be the foot of the perpendicular drawn from the point $R(1, 7, 6)$ on the line passing through the points $(2, -5, 11)$ and $(-6, 7, -5)$. Then the length of the line segment PQ is equal to _____.

Ans. (13)

30. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$, and a vector \vec{c} be such that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times \vec{c} = \hat{i} + 8\hat{j} + 13\hat{k}$. If $\vec{a} \cdot \vec{c} = 13$, then $(24 - \vec{b} \cdot \vec{c})$ is equal to _____.

Ans. (46)



**Download the new ALLEN app
& enroll for Online Programs**

[CLICK HERE TO
DOWNLOAD](#)

Are you targeting **JEE 2025** ?

Ace it with **ALLEN's**
Leader Course

Online Program 18 APRIL '24

Offline Program 24 APRIL '24



ALLEN

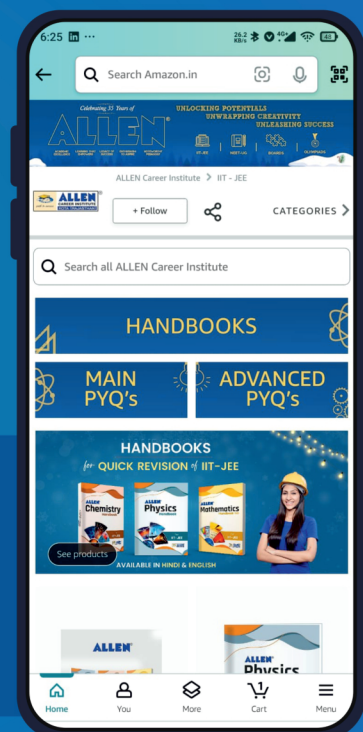
Get The Latest
IIT-JEE Special Books
at Your Door Steps...!!

JOIN THE JOURNEY OF LEARNING

with

SCORE TEST PAPERS | HANDBOOKS
JEE-MAIN PYQ's | JEE-Adv. PYQ's

SHOP NOW



Available in
HINDI & ENGLISH