FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Saturday 06th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is be the sum of areas of all the triangles formed in this process, then:

(1) $P^2=36\sqrt{3}Q$ (2) $P^2=6\sqrt{3}Q$

(3) $P=36\sqrt{3}Q^2$ (4) $P^2=72\sqrt{3}Q$

Ans. (1)

2. Let A = {1, 2, 3, 4, 5}. Let R be a relation on A defined by xRy if and only if $4x \le 5y$. Let m be the number of elements in R and n be the minimum number of elements from A × A that are required to be added to R to make it a symmetric relation. Then m + n is equal to:

(1) 24 (2) 23

(3) 25 (4) 26

Ans. (3)

3. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is:

(1) $\frac{12}{25}$ (2) $\frac{18}{25}$

(3) $\frac{4}{25}$ (4) $\frac{6}{25}$

Ans. (1)

4. Suppose the solution of the differential equation $\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2}{\beta x-2\alpha y-(\beta\gamma-4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is :

(1) $\sqrt{17}$ (2) $\frac{1}{2}$

(3) $\frac{\sqrt{17}}{2}$ (4) 2

Ans. (3)

5. If the locus of the point, whose distances from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is $ax^2 + by^2 + cxy + dx + ey + 170 = 0$, then the value of $a^2 + 2b + 3c + 4d + e$ is equal to:

(1) 5 (2) –27

(3) 37 (4) 437

Ans. (3)

6. $\lim\limits_{n\to\infty}\frac{(1^2-1)(n-1)+(2^2-2)(n-2)+.....+\left((n-1)^2-(n-1)\right)\cdot 1}{(1^3+2^3+.....+n^3)-(1^2+2^2+.....+n^2)}$

is equal to:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. (2)

7. Let $0 \le r \le n$. If ${}^{n+1}C_{r+1} : {}^{n}C_{r} : {}^{n-1}C_{r-1} = 55 : 35 : 21$, then 2n + 5r is equal to:

(1) 60 (2) 62

(3) 50 (4) 55

Ans. (3)

8. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to :

(1) 125 (2) 150

(3) 180 (4) 160

Ans. (2)

9. If z_1, z_2 are two distinct complex number such that $\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\bar{z}_2}\right|=2$, then

(1) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$

(2) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1.

(3) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1.

(4) both z_1 and z_2 lie on the same circle.

Ans. (1)

10. If the function $f(x)=\left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x=\frac{1}{e}$ then :

(1) $e^{\pi} < \pi^e$ (2) $e^{2\pi} < (2\pi)^e$

(3) $e^{\pi} > \pi^e$ (4) $(2e)^{\pi} > \pi^{(2e)}$

Ans. (3)

11. Let $\vec{a}=6\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a is vector such that $|\vec{c}|\geq 6$, $\vec{a}.\vec{c}=6|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 60°, then $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ is equal to:

(1) $\frac{9}{2}(6-\sqrt{6})$ (2) $\frac{3}{2}\sqrt{3}$

(3) $\frac{3}{2}\sqrt{6}$ (4) $\frac{9}{2}(6+\sqrt{6})$

Ans. (4)

12. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315th position in this arrangement is :

(1) NRAGUP (2) NRAGPU

(3) NRAPGU (4) NRAPUG

Ans. (3)

13. Suppose for a differentiable function h, h(0) = 0, h(1) = 1 and h'(0) = h'(1) = 2. If $g(x) = h(e^x)\, e^{h(x)}$, then g'(0) is equal to:

(1) 5 (2) 3

(3) 8 (4) 4

Ans. (4)

14. Let P (α, β, γ) be the image of the point Q(3, –3, 1) in the line $\frac{x-0}{1}=\frac{y-3}{1}=\frac{z-1}{-1}$ and R be the point (2, 5, –1). If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to:

(1) 36 (2) 72

(3) 18 (4) 81

Ans. (4)

15. If P(6, 1) be the orthocentre of the triangle whose vertices are A(5, –2), B(8, 3) and C(h, k), then the point C lies on the circle.

(1) $x^2 + y^2 - 65 = 0$ (2) $x^2 + y^2 - 74 = 0$

(3) $x^2 + y^2 - 61 = 0$ (4) $x^2 + y^2 - 52 = 0$

Ans. (1)

16. Let $f(x)=\frac{1}{7-\sin 5x}$ be a function defined on R. Then the range of the function f(x) is equal to:

(1) $\left[\frac{1}{8},\frac{1}{5}\right]$ (2) $\left[\frac{1}{7},\frac{1}{6}\right]$

(3) $\left[\frac{1}{7},\frac{1}{5}\right]$ (4) $\left[\frac{1}{8},\frac{1}{6}\right]$

Ans. (4)

17. Let $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=\left(\left(\vec{a}\times\left(\hat{i}+\hat{j}\right)\right)\times\hat{i}\right)\times\hat{i}$.

Then the square of the projection of $\vec{a}$ on $\vec{b}$ is :

(1) $\frac{1}{5}$ (2) 2

(3) $\frac{1}{3}$ (4) $\frac{2}{3}$

Ans. (2)

18. If the area of the region

$\left\{(x,y): \frac{a}{x^2} \le y \le \frac{1}{x}, 1 \le x \le 2, 0 < a < 1\right\}$ is

$(\log_e 2) - \frac{1}{7}$ then the value of $7a - 3$ is equal to:

(1) 2 (2) 0

(3) –1 (4) 1

Ans. (3)

19. If $\int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{1}{12} \tan^{-1}(3 \tan x) +$ constant, then the maximum value of asinx + bcosx, is :

(1) $\sqrt{40}$ (2) $\sqrt{39}$

(3) $\sqrt{42}$ (4) $\sqrt{41}$

Ans. (1)

20. If A is a square matrix of order 3 such that det(A) = 3 and $\det(\text{adj}(-4\,\text{adj}(-3\,\text{adj}(3\,\text{adj}((2A)^{-1}))))) = 2^m 3^n$, then m +| 2n is equal to:

(1) 3 (2) 2

(3) 4 (4) 6

Ans. (3)

SECTION-B

21. Let [t] denote the greatest integer less than or equal to t. Let f: [0, ∞) → R be a function defined by $f(x) = \left[\frac{x}{2} + 3\right] - [\sqrt{x}]$. Let S be the set of all points in the interval [0, 8] at which f is not continuous. Then $\sum_{a \in S} a$ is equal to _____.

Ans. (17)

22. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and $x = \pm\frac{4}{\sqrt{3}}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0). If m is the product of the focal distances of the point (x_0, y_0), then $4e^2 + m$ is equal to _____.

NTA Ans. (61)

23. If $S(x) = (1 + x) + 2(1 + x)^2 + 3(1 + x)^3 + \ldots + 60(1 + x)^{60}$, $x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where $a, b \in N$, then (a + b) equal to _____

Ans. (3660)

24. Let [t] denote the largest integer less than or equal to t. If

$$\int_0^3 \left([x^2] + \left[\frac{x^2}{2}\right]\right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7},$$

where $a, b, c \in z$, then a + b + c is equal to _____

Ans. (23)

25. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where gcd(m, n) = 1, then n – m is equal to _____.

Ans. (71)

26. In a triangle ABC, BC = 7, AC = 8, AB = $\alpha \in N$ and $\cos A = \frac{2}{3}$. If $49\cos(3C) + 42 = \frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to _____

Ans. (39)

27. If the shortest distance between the lines $\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to _____.

Ans. (43)

28. Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$.

If $U_n = \alpha^n + \beta^n$, then $\frac{U_{10} + \sqrt{12}U_9}{2U_8}$ is equal to _____.

Ans. (4)

29. If the system of equations

$2x + 7y + \lambda z = 3$

$3x + 2y + 5z = 4$

$x + \mu y + 32z = -1$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to _____ :

Ans. (38)

30. If the solution y(x) of the given differential equation $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$ passes through the point $\left(\frac{\pi}{2}, 0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to _____.

Ans. (3)