FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Friday 05th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS **TEST PAPER WITH ANSWER**

SECTION-A

1. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ from the point (7, 8, 9). Then $d^2 + 6$ is equal to :

(1) 72 (2) 69

(3) 75 (4) 78

Ans. (3)

2. Let a rectangle ABCD of sides 2 and 4 be inscribed in another rectangle PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a + b)^2$ is equal to :

(1) 72 (2) 60

(3) 80 (4) 64

Ans. (1)

3. Let two straight lines drawn from the origin O intersect the line 3x + 4y = 12 at the points P and Q such that ΔOPQ is an isosceles triangle and ∠POQ = 90°. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to l is :

(1) 44 (2) 48

(3) 46 (4) 42

Ans. (3)

4. If y = y(x) is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x)$, $y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to :

(1) $e^{-\pi/8}$ (2) $e^{-\pi/4}$

(3) $e^{\pi/4}$ (4) $e^{\pi/8}$

Ans. (2)

5. For the function

$f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x)$, where $x \in \left[0, \frac{\pi}{2}\right]$,

consider the following two statements :

(I) f is increasing in $\left(0, \frac{\pi}{2}\right)$.

(II) f′ is decreasing in $\left(0, \frac{\pi}{2}\right)$.

Between the above two statements,

(1) only (I) is true.

(2) only (II) is true.

(3) neither (I) nor (II) is true.

(4) both (I) and (II) are true.

Ans. (4)

6. If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

(1) 49 (2) 45

(3) 47 (4) 51

Ans. (3)

7. Let A = {1, 3, 7, 9, 11} and B = {2, 4, 5, 7, 8, 10, 12}. Then the total number of one-one maps $f : A \to B$, such that f (1) + f(3) = 14, is :

(1) 180 (2) 120

(3) 480 (4) 240

Ans. (4)

8. If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in R$, is continuous at x = 0, then f(0) is equal to :

(1) 2 (2) –2

(3) 4 (4) –4

Ans. (4)

9. The integral $\int_0^{\frac{\pi}{4}} \frac{136\sin x}{3\sin x + 5\cos x}dx$ is equal to :

(1) $3\pi - 50 \log_e 2 + 20 \log_e 5$

(2) $3\pi - 25 \log_e 2 + 10 \log_e 5$

(3) $3\pi - 10 \log_e\left(2\sqrt{2}\right) + 10 \log_e 5$

(4) $3\pi - 30 \log_e 2 + 20 \log_e 5$

Ans. (1)

10. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$. The probability of this equation having repeated roots is :

(1) $\frac{3}{256}$ (2) $\frac{1}{128}$

(3) $\frac{1}{64}$ (4) $\frac{3}{128}$

Ans. (3)

11. Let A and B be two square matrices of order 3 such that $|A| = 3$ and $|B| = 2$.
Then $|A^T A(\text{adj}(2A))^{-1} (\text{adj}(4B))(\text{adj}(AB))^{-1}AA^T|$ is equal to :

(1) 64 (2) 81

(3) 32 (4) 108

Ans. (1)

12. Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point (3, 2) and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point (5, 5) is :

(1) $2\sqrt{2}$ (2) 5

(3) $4\sqrt{2}$ (4) 4

Ans. (4)

13. Let the line $2x + 3y - k = 0$, $k > 0$, intersect the x-axis and y-axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is $x^2 + y^2 - 3x - 2y = 0$ and the length of the latus rectum of the ellipse $x^2 + 9y^2 = k^2$ is $\frac{m}{n}$, where m and n are coprime, then $2m + n$ is equal to

(1) 10 (2) 11

(3) 13 (4) 12

Ans. (2)

14. Consider the following two statements :

Statement I : For any two non-zero complex numbers z_1, z_2

$\left(|z_1|+|z_2|\right)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2\left(|z_1|+|z_2|\right)$ and

Statement II : If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$, then

$\frac{a^2}{y-z}+\frac{b^2}{z-x}+\frac{c^2}{x-y}=1$.

Between the above two statements,

(1) both Statement I and Statement II are incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Statement I is correct but Statement II is incorrect.

(4) both Statement I and Statement II are correct.

Ans. (3)

15. Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$.

Then $\cos\theta$ is equal to :

(1) $\frac{4}{(3\sqrt{6}-2)}$ (2) $\frac{6-\sqrt{6}}{(3\sqrt{6}-2)}$

(3) $\frac{6+\sqrt{6}}{(3\sqrt{6}+2)}$ (4) $\frac{4}{(3\sqrt{6}+2)}$

Ans. (1)

16. If $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{99}+\sqrt{100}}=m$ and $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+...+\frac{1}{99\cdot 100}=n$, then the point (m, n) lies on the line

(1) $11(x-1)-100(y-2)=0$

(2) $11(x-2)-100(y-1)=0$

(3) $11(x-1)-100y=0$

(4) $11x-100y=0$

Ans. (4)

17. Let $f(x)= x^5 + 2x^3 + 3x + 1$, $x \in R$, and g(x) be a function such that $g(f(x))=x$ for all $x \in R$. Then $\frac{g(7)}{g'(7)}$ is equal to :

(1) 7 (2) 42

(3) 1 (4) 14

Ans. (4)

18. If A(l, –1, 2), B(5, 7, –6), C(3, 4, –10) and D(–l, –4, –2) are the vertices of a quadrilateral ABCD, then its area is :

(1) $12\sqrt{29}$ (2) $24\sqrt{29}$

(3) $24\sqrt{7}$ (4) $48\sqrt{7}$

Ans. (1)

19. The value of $\int_{-\pi}^{\pi} \frac{2y(1+\sin y)}{1+\cos^2 y}dy$ is :

(1) π^2 (2) $\frac{\pi^2}{2}$

(3) $\frac{\pi}{2}$ (4) $2\pi^2$

Ans. (1)

20. If the line $\frac{2-x}{3}=\frac{3y-2}{4\lambda+1}=4-z$ makes a right angle with the line $\frac{x+3}{3\mu}=\frac{1-2y}{6}=\frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to :

(1) 13 (2) 4

(3) 5 (4) 6

Ans. (4)

SECTION-B

21. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to__________.

Ans. (56)

22. If the constant term in the expansion of $(1 + 2x - 3x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is p, then 108p is equal to

Ans. (54)

23. The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is __________.

Ans. (72)

NTA Ans. (198)

24. The number of ways of getting a sum 16 on throwing a dice four times is __________.

Ans. (125)

25. If S = {a ∈ R : |2a – 1| = 3[a]+2{a}}, where [t] denotes the greatest integer less than or equal to t and {t} represents the fractional part of t, then $72\sum_{a\in S} a$ is equal to __________.

Ans. (18)

26. Let f be a differentiable function in the interval (0, ∞) such that f(l) = 1 and $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$ for each x > 0. Then 2 f(2) + 3 f(3) is equal to __________.

Ans. (24)

27. Let $a_1, a_2, a_3, \ldots$ be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to __________.

Ans. (910)

28. Let $\vec{a}=\hat{i}-3\hat{j}+7\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a}+2\vec{b})\times\vec{c}=3(\vec{c}\times\vec{a})$. If $\vec{a}\cdot\vec{c}=130$, then $\vec{b}\cdot\vec{c}$ is equal to __________.

Ans. (30)

29. The number of distinct real roots of the equation |x| |x + 2| – 5|x + 1| – 1 = 0 is __________.

Ans. (3)

30. Suppose AB is a focal chord of the parabola $y^2 = 12x$ of length l and slope $m<\sqrt{3}$. If the distance of the chord AB from the origin is d, then ld^2 is equal to __________.

Ans. (108)