FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Friday 05th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH ANSWER

SECTION-A

1. Let $f: [-1, 2] \rightarrow R$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where [t] denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

(1) 6 (2) 3
(3) 4 (4) 5

Ans. (3)

2. The differential equation of the family of circles passing the origin and having center at the line $y = x$ is :

(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$
(2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$
(3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$
(4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

Ans. (3)

3. Let $S_1 = \{z \in C : |z| \le 5\}$,

$$S_2 = \left\{z \in C : \operatorname{Im}\left(\frac{z + 1 - \sqrt{3}\,i}{1 - \sqrt{3}\,i}\right) \ge 0\right\} \text{ and}$$

$S_3 = \{z \in C : \operatorname{Re}(z) \ge 0\}$. Then the area of region $S_1 \cap S_2 \cap S_3$ is

(1) $\frac{125\pi}{6}$ (2) $\frac{125\pi}{24}$
(3) $\frac{125\pi}{4}$ (4) $\frac{125\pi}{12}$

Ans. (4)

4. The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is :

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$
(3) 1 (4) $\frac{4}{3}$

Ans. (4)

5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50^{th} word is :

(1) OBBHJ (2) HBBJO
(3) OBBJH (4) JBBOH

Ans. (3)

6. Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{c}$ be three vectors such that $\left(\vec{c} + \hat{i}\right) \times \left(\vec{a} + \vec{b} + \hat{i}\right) = \vec{a} \times \left(\vec{c} + \hat{i}\right)$. $\vec{a}.\vec{c} = -29$, then $\vec{c}.\left(-2\hat{i} + \hat{j} + \hat{k}\right)$ is equal to :

(1) 10 (2) 5
(3) 15 (4) 12

Ans. (2)

7. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to :

(1) 110 (2) 105
(3) 124 (4) 121

Ans. (3)

8. Let A(–1, 1) and B(2, 3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is $ax + by = 15$, then $5a + 2b$ is :

(1) $-\frac{12}{5}$ (2) $-\frac{6}{5}$
(3) 4 (4) 6

Ans. (1)

9. Let (α, β, γ) be the image of the point (8, 5, 7) in the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{5}$. Then α + β + γ is equal to

(1) 16 (2) 18

(3) 14 (4) 20

Ans. (3)

10. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x}+\frac{2x}{\sqrt[3]{5}}\right)^{12}$, $x \neq 0$, is $\alpha \times 2^8 \times \sqrt[5]{3}$, then 25α is equal to :

(1) 639 (2) 724

(3) 693 (4) 742

Ans. (3)

11. Let f, g : R → R be defined as : $f(x) = |x - 1|$ and $g(x)=\begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}$. Then the function $f(g(x))$ is

(1) neither one-one nor onto.

(2) one-one but not onto.

(3) both one-one and onto.

(4) onto but not one-one.

Ans. (1)

12. Let the circle $C_1 : x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at (–1, 0) and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

(1) 2 (2) 1

(3) 6 (4) 4

Ans. (1)

13. Let the set S = {2, 4, 8, 16, ..., 512} be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to :

(1) 1680 (2) 1520

(3) 1710 (4) 1640

Ans. (1)

14. The values of m, n, for which the system of equations

$x + y + z = 4,$

$2x + 5y + 5z = 17,$

$x + 2y + mz = n$

has infinitely many solutions, satisfy the equation :

(1) $m^2 + n^2 - m - n = 46$

(2) $m^2 + n^2 + m + n = 64$

(3) $m^2 + n^2 + mn = 68$

(4) $m^2 + n^2 - mn = 39$

Ans. (4)

15. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set {1, 2, 3, 4, 5, 6}. If the probability of this equation having one real root bigger than the other is p, then 216p equals :

(1) 57 (2) 38

(3) 19 (4) 76

Ans. (2)

16. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies :

(1) r = 1 (2) $r^2 - 8r + 8 = 0$

(3) $2r^2 - 4r + 1 = 0$ (4) $2r^2 - 8r + 7 = 0$

Ans. (2)

17. Let $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0. If $\int_0^1 (1-x^{10})^{20}dx = a \times \beta(b,c)$, then 100(a + b + c) equals ______.

(1) 1021 (2) 1120

(3) 2012 (4) 2120

Ans. (4)

18. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$.

If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then det(AB) is equal to :

(1) 343 (2) 125

(3) 64 (4) 216

Ans. (4)

19. If $y(\theta) = \dfrac{2\cos\theta + \cos 2\theta}{\cos 3\theta + 4\cos 2\theta + 5\cos\theta + 2}$,

then at $\theta = \dfrac{\pi}{2}$, y'' + y' + y is equal to:

(1) $\dfrac{3}{2}$ (2) 1

(3) $\dfrac{1}{2}$ (4) 2

Ans. (4)

20. For $x \geq 0$, the least value of K, for which $4^{1+x} + 4^{1-x}$, $\dfrac{K}{2}$, $16^{x} + 16^{-x}$ are three consecutive terms of an A.P. is equal to :

(1) 10 (2) 4

(3) 8 (4) 16

Ans. (1)

SECTION-B

21. Let the mean and the standard deviation of the probability distribution

X	α	1	0	-3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to ______.

Ans. (5)

22. Let y = y(x) be the solution of the differential equation $\dfrac{dy}{dx} + \dfrac{2x}{(1+x^2)^2} y = xe^{\frac{1}{(1+x^2)}}$; y(0) = 0.

Then the area enclosed by the curve $f(x) = y(x)e^{-\frac{1}{(1+x^2)}}$ and the line $y - x = 4$ is ____.

Ans. (18)

23. The number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x-1)^2 = 0$, where $-\pi \leq x \leq \pi$, is

Ans. (2)

24. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\dfrac{x+2}{-3} = \dfrac{y-2}{4} = \dfrac{z-5}{2}$ and $\dfrac{x+2}{-1} = \dfrac{y+6}{2} = \dfrac{z-1}{0}$.

Then $(\alpha - \beta)^2$ is equal to _____.

Ans. (25)

25. If

$$1 + \frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}} + \frac{5-2\sqrt{6}}{18} + \frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}} + \frac{49-20\sqrt{6}}{180} + \ldots$$

upto $\infty = 2\left(\sqrt{\dfrac{b}{a}}+1\right)\log_e\left(\dfrac{a}{b}\right)$, where a and b are integers with gcd(a, b) = 1, then 11a + 18b is equal to ______.

Ans. (76)

26. Let a > 0 be a root of the equation $2x^2 + x - 2 = 0$. If $\lim\limits_{x \to \frac{1}{a}} \dfrac{16\left(1-\cos(2+x-2x^2)\right)}{(1-ax^2)} = \alpha + \beta\sqrt{17}$, where $\alpha, \beta \in Z$ then $\alpha + \beta$ is equal to ______.

Ans. (170)

27. If $f(t)=\int\limits_0^{\pi}\frac{2x\,dx}{1-\cos^2 t\sin^2 x}$, $0<t<\pi$, then the value of $\int\limits_0^{\frac{\pi}{2}}\frac{\pi^2 dt}{f(t)}$ equals _____.

Ans. (1)

28. Let the maximum and minimum values of $\left(\sqrt{8x-x^2-12}-4\right)^2+(x-7)^2$, $x\in R$ be M and m respectively. Then M^2-m^2 is equal to _____.

Ans. (1600)

29. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is ______.

Ans. (10)

30. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is _____.

Ans. (3)