FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS **TEST PAPER WITH ANSWER**

SECTION-A

1. Let $f : R \to R$ be a function given by

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2} & , \; x<0\\ \alpha & , \; x=0\\ \dfrac{\beta\sqrt{1-\cos x}}{x} & , \; x>0\end{cases}$$

, where $\alpha, \beta \in R$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to :

(1) 48 (2) 12

(3) 3 (4) 6

Ans. (2)

2. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\frac{4}{17}$ (2) $\frac{5}{18}$

(3) $\frac{7}{18}$ (4) $\frac{5}{16}$

Ans. (2)

3. The vertices of a triangle are A(–1, 3), B(–2, 2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

(1) $x - y - (2+\sqrt{2}) = 0$

(2) $-x + y - (2-\sqrt{2}) = 0$

(3) $x + y - (2-\sqrt{2}) = 0$

(4) $x + y + (2-\sqrt{2}) = 0$

Ans. (3)

4. If the solution $y = y(x)$ of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\frac{\pi}{4}$, then $y(0)$ is equal to :

(1) $-\frac{\pi}{12}$ (2) 0

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. (3)

5. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$, where $\gcd(m, n) = 1$. Then $m + n$ is equal to :

(1) 182 (2) 217

(3) 195 (4) 201

Ans. (4)

6. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}\left(\ell\sqrt{5}+m\right) - n\log_e\left(1+\sqrt{5}\right)$, where $\ell, m, n \in N$. Then $\ell + m + n$ is equal to

(1) 32 (2) 30

(3) 29 (4) 31

Ans. (2)

7. If the system of equations

$$x+\left(\sqrt{2}\sin\alpha\right)y+\left(\sqrt{2}\cos\alpha\right)z=0$$

$$x + (\cos\alpha)y + (\sin\alpha)z = 0$$

$$x + (\sin\alpha)y - (\cos\alpha)z = 0$$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to :

(1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{24}$

(3) $\frac{5\pi}{24}$ (4) $\frac{11\pi}{24}$

Ans. (3)

8. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points $P_6, P_7, \ldots, P_{11}$ on the side BC and 7 points $P_{12}, P_{13}, \ldots, P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $P_1, P_2, \ldots, P_{18}$ as vertices, is :

(1) 776 (2) 751

(3) 796 (4) 771

Ans. (2)

9. Let $f(x) = \begin{cases} -2, & -2 \le x \le 0 \\ x-2, & 0 < x \le 2 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$.

Then $\int_{-2}^{2} h(x)dx$ is equal to :

(1) 2 (2) 4

(3) 1 (4) 6

Ans. (1)

10. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$ is equal to :

(1) 3133 (2) 633

(3) 931 (4) 6131

Ans. (1)

11. Let a unit vector which makes an angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$.

Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is :

(1) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

(2) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

(3) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(4) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

Ans. (4)

12. Let the first three terms 2, p and q, with $q \neq 2$, of a G.P. be respectively the 7th, 8th and 13th terms of an A.P. If the 5th term of the G.P. is the nth term of the A.P., then n is equal to

(1) 151 (2) 169

(3) 177 (4) 163

Ans. (4)

13. Let $a, b \in R$. Let the mean and the variance of 6 observations –3, 4, 7, –6, a, b be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is :

(1) $\frac{13}{3}$ (2) $\frac{16}{3}$

(3) $\frac{11}{3}$ (4) $\frac{14}{3}$

Ans. (1)

14. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

(1) $2x^2 + 11x + 12 = 0$ (2) $4x^2 + 14x + 12 = 0$

(3) $x^2 + 10x + 16 = 0$ (4) $x^2 + 8x + 12 = 0$

Ans. (4)

15. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to :

(1) 6 (2) 4

(3) 8 (4) 2

Ans. (2)

16. Let the point, on the line passing through the points P(1, –2, 3) and Q(5, –4, 7), farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 155 (2) 150

(3) 160 (4) 165

Ans. (1)

17. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum(x_i^2 + y_i^2)$ is equal to :

(1) 148 (2) 156

(3) 160 (4) 152

Ans. (4)

18. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to :

(1) 97 (2) 100

(3) 95 (4) 98

Ans. (1)

19. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in R$. Consider a function $g(x)$ such that $(g o f)(x) = x$ for all $x \in R$. Then the value of $8g'(2)$ is :

(1) 16 (2) 4

(3) 8 (4) 2

Ans. (1)

20. Let $\alpha \in (0, \infty)$ and $A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

If $\det(\text{adj}(2A - A^T).\text{adj}(A - 2A^T)) = 2^8$, then $(\det(A))^2$ is equal to :

(1) 1 (2) 49

(3) 16 (4) 36

Ans. (3)

SECTION-B

21. If $\lim\limits_{x\to 1}\frac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$, where $\gcd(m, n) = 1$, then $8m + 12n$ is equal to _____

Ans. (100)

22. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to _____

Ans. (45)

23. Let the solution $y = y(x)$ of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to _____

Ans. (7)

24. If the shortest distance between the lines $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ and $\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$ and $\int_0^k [x^2]dx = \alpha - \sqrt{\alpha}$, where $[x]$ denotes the greatest integer function, then $6\alpha^3$ is equal to _____

Ans. (48)

25. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is -3. If the points (x, y) satisfying $A^2 + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is ℓ, then $e^4 + \ell^4$ is equal to ______

Ans. (Bouns)
NTA Ans. (25)

26. Let $a = 1 + \frac{^2C_2}{3!} + \frac{^3C_2}{4!} + \frac{^4C_2}{5!} + \ldots,$

$$b = 1 + \frac{^1C_0 + {^1C_1}}{1!} + \frac{^2C_0 + {^2C_1} + {^2C_2}}{2!} + \frac{^3C_0 + {^3C_1} + {^3C_2} + {^3C_3}}{3!} + \ldots$$

Then $\frac{2b}{a^2}$ is equal to ______

Ans. (8)

27. Let A be a 3×3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of det(A) is _____

Ans. (27)

28. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to ____

Ans. (72)

29. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i} + 2\hat{j} - 7\hat{k}$, $\overrightarrow{BC} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}$, $d > 0$. Then the square of the length of the largest side of the triangle ABC is

Ans. (54)

30. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1 + \sin x \cos x} dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$, where $a, b \in N$, then $a + b$ is equal to _____

Ans. (8)