

FINAL JEE-MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04th April, 2024)

TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function given by

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0, \text{ where } \alpha, \beta \in \mathbb{R}. \\ \frac{\beta \sqrt{1 - \cos x}}{x}, & x > 0 \end{cases}$$

f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to :

- (1) 48 (2) 12
(3) 3 (4) 6

Ans. (2)

2. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

- (1) $\frac{4}{17}$ (2) $\frac{5}{18}$
(3) $\frac{7}{18}$ (4) $\frac{5}{16}$

Ans. (2)

3. The vertices of a triangle are $A(-1, 3)$, $B(-2, 2)$ and $C(3, -1)$. A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

- (1) $x - y - (2 + \sqrt{2}) = 0$
(2) $-x + y - (2 - \sqrt{2}) = 0$
(3) $x + y - (2 - \sqrt{2}) = 0$
(4) $x + y + (2 - \sqrt{2}) = 0$

Ans. (3)

4. If the solution $y = y(x)$ of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\frac{\pi}{4}$, then $y(0)$ is equal to :

- (1) $-\frac{\pi}{12}$ (2) 0
(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Ans. (3)

5. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$ be $\frac{m}{n}$, where $\gcd(m, n) = 1$. Then $m + n$ is equal to :

- (1) 182 (2) 217
(3) 195 (4) 201

Ans. (4)

6. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $(\frac{1}{2}, 2)$. Let the area of the region enclosed by these curves be $\frac{1}{24}(\ell\sqrt{5} + m) - n \log_e(1 + \sqrt{5})$, where $\ell, m, n \in \mathbb{N}$. Then $\ell + m + n$ is equal to

- (1) 32 (2) 30
(3) 29 (4) 31

Ans. (2)

7. If the system of equations $x + (\sqrt{2} \sin \alpha)y + (\sqrt{2} \cos \alpha)z = 0$
 $x + (\cos \alpha)y + (\sin \alpha)z = 0$
 $x + (\sin \alpha)y - (\cos \alpha)z = 0$

has a non-trivial solution, then $\alpha \in (0, \frac{\pi}{2})$ is equal to :

- (1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{24}$
(3) $\frac{5\pi}{24}$ (4) $\frac{11\pi}{24}$

Ans. (3)



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8. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points P_6, P_7, \dots, P_{11} on the side BC and 7 points $P_{12}, P_{13}, \dots, P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points P_1, P_2, \dots, P_{18} as vertices, is :
- (1) 776 (2) 751
 (3) 796 (4) 771

Ans. (2)

9. Let $f(x) = \begin{cases} -2, & -2 \leq x \leq 0 \\ x-2, & 0 < x \leq 2 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$.

Then $\int_{-2}^2 h(x) dx$ is equal to :

- (1) 2 (2) 4
 (3) 1 (4) 6

Ans. (1)

10. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$ is equal to :

- (1) 3133 (2) 633
 (3) 931 (4) 6131

Ans. (1)

11. Let a unit vector which makes an angle of 60° with $2\hat{i} + 2\hat{j} - \hat{k}$ and an angle of 45° with $\hat{i} - \hat{k}$ be \vec{C} .

Then $\vec{C} + \left(-\frac{1}{2}\hat{i} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{\sqrt{2}}{3}\hat{k}\right)$ is :

- (1) $-\frac{\sqrt{2}}{3}\hat{i} + \frac{\sqrt{2}}{3}\hat{j} + \left(\frac{1}{2} + \frac{2\sqrt{2}}{3}\right)\hat{k}$
 (2) $\frac{\sqrt{2}}{3}\hat{i} + \frac{1}{3\sqrt{2}}\hat{j} - \frac{1}{2}\hat{k}$
 (3) $\left(\frac{1}{\sqrt{3}} + \frac{1}{2}\right)\hat{i} + \left(\frac{1}{\sqrt{3}} - \frac{1}{3\sqrt{2}}\right)\hat{j} + \left(\frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{3}\right)\hat{k}$
 (4) $\frac{\sqrt{2}}{3}\hat{i} - \frac{1}{2}\hat{k}$

Ans. (4)

12. Let the first three terms 2, p and q, with $q \neq 2$, of a G.P. be respectively the $7^{\text{th}}, 8^{\text{th}}$ and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to

- (1) 151 (2) 169
 (3) 177 (4) 163

Ans. (4)

13. Let $a, b \in \mathbb{R}$. Let the mean and the variance of 6 observations $-3, 4, 7, -6, a, b$ be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is :

- (1) $\frac{13}{3}$ (2) $\frac{16}{3}$
 (3) $\frac{11}{3}$ (4) $\frac{14}{3}$

Ans. (1)

14. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

- (1) $2x^2 + 11x + 12 = 0$ (2) $4x^2 + 14x + 12 = 0$
 (3) $x^2 + 10x + 16 = 0$ (4) $x^2 + 8x + 12 = 0$

Ans. (4)

15. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0, z \in \mathbb{C}$. Then $4(\alpha^2 + \beta^2)$ is equal to :

- (1) 6 (2) 4
 (3) 8 (4) 2

Ans. (2)

16. Let the point, on the line passing through the points $P(1, -2, 3)$ and $Q(5, -4, 7)$, farther from the origin and at a distance of 9 units from the point P, be (α, β, γ) . Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

- (1) 155 (2) 150
 (3) 160 (4) 165

Ans. (1)

25. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is -3 . If the points (x, y) satisfying $A^2 + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the x -axis, eccentricity is e and the length of the latus rectum is ℓ , then $e^4 + \ell^4$ is equal to _____

Ans. (Bouns)

NTA Ans. (25)

26. Let $a = 1 + \frac{{}^2C_2}{{}^3!} + \frac{{}^3C_2}{{}^4!} + \frac{{}^4C_2}{{}^5!} + \dots$,
 $b = 1 + \frac{{}^1C_0 + {}^1C_1}{{}^1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{{}^2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{{}^3!} + \dots$

Then $\frac{2b}{a^2}$ is equal to _____

Ans. (8)

27. Let A be a 3×3 matrix of non-negative real

elements such that $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Then the

maximum value of $\det(A)$ is _____

Ans. (27)

28. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p , then $10p^2$ is equal to _____

Ans. (72)

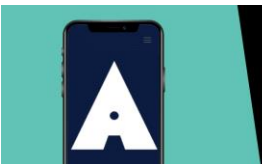
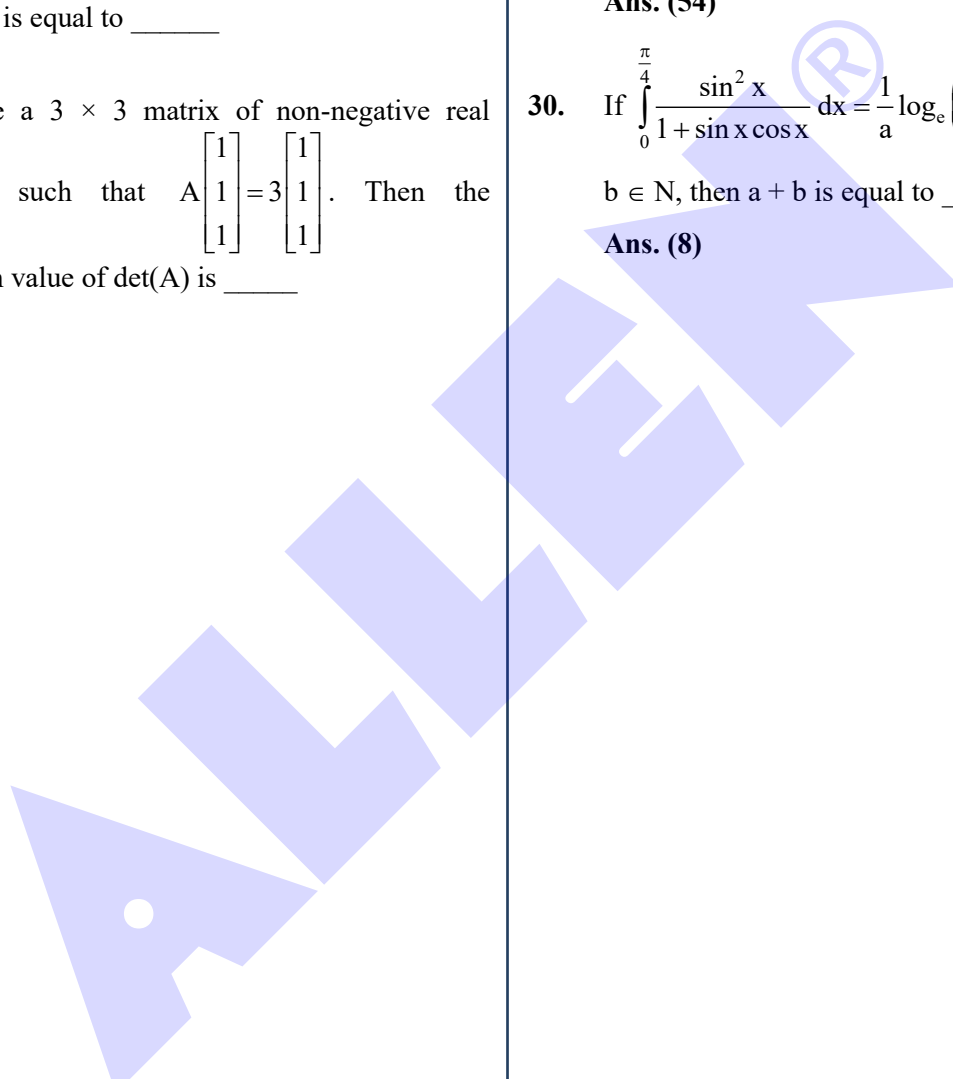
29. Let $\triangle ABC$ be a triangle of area $15\sqrt{2}$ and the vectors $\vec{AB} = \hat{i} + 2\hat{j} - 7\hat{k}$, $\vec{BC} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\vec{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}$, $d > 0$. Then the square of the length of the largest side of the triangle ABC is

Ans. (54)

30. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1 + \sin x \cos x} dx = \frac{1}{a} \log_e \left(\frac{a}{3} \right) + \frac{\pi}{b\sqrt{3}}$, where $a,$

$b \in \mathbb{N}$, then $a + b$ is equal to _____

Ans. (8)



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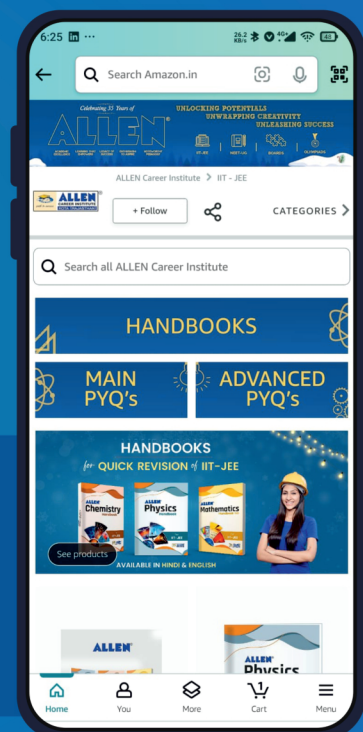
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