FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH ANSWER**

SECTION-A

1. If the function $f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}} & , x\neq 0\\ a\log_e 2\log_e 3 & , x=0\end{cases}$ is continuous at x = 0, then the value of a^2 is equal to

(1) 968 (2) 1152
(3) 746 (4) 1250

Ans. (2)

2. If $\lambda > 0$, let θ be the angle between the vectors $\vec{a}=\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$. If the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to

(1) 25 (2) 20
(3) 50 (4) 40

Ans. (1)

3. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line x + y = 2 intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope –1. Then, a distance (in units) between the chord PQ and the chord MN is

(1) $2-\sqrt{3}$ (2) $3-\sqrt{2}$
(3) $\sqrt{2}-1$ (4) $\sqrt{2}+1$

Ans. (2)

4. Let a relation R on $\mathbb{N}\times\mathbb{N}$ be defined as :
$(x_1,y_1)\ R\ (x_2,y_2)$ if and only if $x_1 \le x_2$ or $y_1 \le y_2$
Consider the two statements :
(I) R is reflexive but not symmetric.
(II) R is transitive
Then which one of the following is true ?
(1) Only (II) is correct.
(2) Only (I) is correct.
(3) Both (I) and (II) are correct.
(4) Neither (I) nor (II) is correct.

Ans. (2)

5. Let three real numbers a,b,c be in arithmetic progression and a + 1, b, c + 3 be in geometric progression. If a > 10 and the arithmetic mean of a,b and c is 8, then the cube of the geometric mean of a,b and c is

(1) 120 (2) 312
(3) 316 (4) 128

Ans. (1)

6. Let $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $B = I + \text{adj}(A) + (\text{adj } A)^2+\ldots+(\text{adj } A)^{10}$. Then, the sum of all the elements of the matrix B is :

(1) –110 (2) 22
(3) –88 (4) –124

Ans. (3)

7. The value of $\dfrac{1\times 2^2+2\times 3^2+\ldots+100\times(101)^2}{1^2\times 2+2^2\times 3+\ldots+100^2\times 101}$ is

(1) $\dfrac{306}{305}$ (2) $\dfrac{305}{301}$
(3) $\dfrac{32}{31}$ (4) $\dfrac{31}{30}$

Ans. (2)

8. Let $f(x)=\int_0^x\left(t+\sin\left(1-e^t\right)\right)dt, x\in\mathbb{R}$.

Then $\lim\limits_{x\to 0}\dfrac{f(x)}{x^3}$ is equal to

(1) $\dfrac{1}{6}$ (2) $-\dfrac{1}{6}$
(3) $-\dfrac{2}{3}$ (4) $\dfrac{2}{3}$

Ans. (2)

9. The area (in sq. units) of the region described by $\{(x,y) : y^2 \le 2x, \text{ and } y \ge 4x - 1\}$ is

(1) $\frac{11}{32}$ (2) $\frac{8}{9}$

(3) $\frac{11}{12}$ (4) $\frac{9}{32}$

Ans. (4)

10. The area (in sq. units) of the region $S = \{z \in \mathbb{C}; |z-1| \le 2; (z+\bar{z}) + i(z-\bar{z}) \le 2, \text{Im}(z) \ge 0\}$ is

(1) $\frac{7\pi}{3}$ (2) $\frac{3\pi}{2}$

(3) $\frac{17\pi}{8}$ (4) $\frac{7\pi}{4}$

Ans. (2)

11. If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx$ is $\frac{2}{\pi}$.

Then, a value of α is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (2)

12. Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to

(1) 44 (2) 42

(3) 24 (4) 38

Ans. (2)

13. If the coefficients of x^4, x^5 and x^6 in the expansion of $(1 + x)^n$ are in the arithmetic progression, then the maximum value of n is :

(1) 14 (2) 21

(3) 28 (4) 7

Ans. (1)

14. Consider a hyperbola H having centre at the origin and foci and the x-axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq. units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is

(1) $\frac{28}{3}$ (2) $\frac{14}{3}$

(3) $\frac{10}{3}$ (4) $\frac{11}{3}$

Ans. (1)

15. If the mean of the following probability distribution of a random variable X;

X	0	2	4	6	8
P(X)	a	2a	a + b	2b	3b

is $\frac{46}{9}$, then the variance of the distribution is

(1) $\frac{581}{81}$ (2) $\frac{566}{81}$

(3) $\frac{173}{27}$ (4) $\frac{151}{27}$

Ans. (2)

16. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at (4,1). Then, which of the following point lies on the line passing through the points P and Q ?

(1) (3,–3) (2) $\left(\frac{3}{2}, -16\right)$

(3) (2,–9) (4) $\left(\frac{1}{2}, -20\right)$

Ans. (4)

17. Given the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in [–1,1] such that

$\cos^{-1}x - \sin^{-1}y = \alpha, \frac{-\pi}{2} \le \alpha \le \pi$.

Then, the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is

(1) –1 (2) 0

(3) $\frac{-1}{2}$ (4) $\frac{1}{2}$

Ans. (2)

18. Let y = y(x) be the solution of the differential equation

$(x^2 + 4)^2dy + (2x^3y + 8xy - 2)dx = 0$. If y(0) = 0, then y(2) is equal to

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{16}$

(3) 2π (4) $\frac{\pi}{32}$

Ans. (4)

19. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in \mathbb{R}$. If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a}.\vec{d} = 1$, then $(\vec{a} \times \vec{b}).\vec{c}$ is equal to

(1) 9 (2) 6

(3) 3 (4) 11

Ans. (4)

20. Let P the point of intersection of the lines $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$ and $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$.

Then, the shortest distance of P from the line $4x = 2y = z$ is

(1) $\frac{5\sqrt{14}}{7}$ (2) $\frac{\sqrt{14}}{7}$

(3) $\frac{3\sqrt{14}}{7}$ (4) $\frac{6\sqrt{14}}{7}$

Ans. (3)

SECTION-B

21. Let S = $\{\sin^2 2\theta : (\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots$\}$. If α and β be the smallest and largest elements of the set S, respectively, then $3((\alpha - 2)^2 + (\beta - 1)^2)$ equals…..

Ans. (4)

22. If $\int \operatorname{cosec}^5 x\,dx = \alpha \cot x \operatorname{cosec} x\left(\operatorname{cosec}^2 x + \frac{3}{2}\right) + \beta \log_e\left|\tan\frac{x}{2}\right| + C$

where $\alpha, \beta \in \mathbb{R}$ and C is constant of integration, then the value of $8(\alpha + \beta)$ equals …..

Ans. (1)

23. Let $f : \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function such that f(0) = 0, f(1) = 1, f(2) = –1, f(3) = 2 and f(4) = –2. Then, the minimum number of zeros of (3f' f'' + ff''') (x) is …..

Ans. (5)

24. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of

$f, \underbrace{(f \text{ o } f \text{ o } f \text{ o} \ldots \text{o } f)}_{10 \text{ times}}(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha+1}$ is equal to …..

Ans. (1024)

25. Let A be a 2 × 2 symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}$ and the determinant of A be 1.

If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2 × 2, then α + β equals …..

Ans. (5)

26. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is …..

Ans. (5626)

27. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x-y| \le 2)$ is p, then 3^9p equals……

Ans. (8288)

28. Consider a triangle ABC having the vertices A(1,2), B(α,β) and C(γ,δ) and angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If the points B and C lie on the line $y = x + 4$, then $\alpha^2 + \gamma^2$ is equal to …..

Ans. (14)

29. Consider a line L passing through the points P(1,2,1) and Q(2,1,–1). If the mirror image of the point A(2,2,2) in the line L is (α,β,γ), then $\alpha + \beta + 6\gamma$ is equal to …..

Ans. (6)

30. Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2\, dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}, \gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals …..

Ans. (31)