

FINAL JEE-MAIN EXAMINATION – APRIL, 2023

(Held On Monday 10th April, 2023)

TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS	TEST PAPER WITH SOLUTION
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SECTION-A

1. Let f be a continuous function satisfying $\int_0^{t^2} (f(x) + x^2) dx = \frac{4}{3}t^3, \forall t > 0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to :

(1) $\pi\left(1 - \frac{\pi^3}{16}\right)$

(2) $-\pi^2\left(1 + \frac{\pi^2}{16}\right)$

(3) $-\pi\left(1 + \frac{\pi^3}{16}\right)$

(4) $\pi^2\left(1 - \frac{\pi^2}{16}\right)$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\int_0^{t^2} (f(x) + x^2) dx = \frac{4}{3}t^3, \forall t > 0$

$$(f(t^2) + t^4) = 2t$$

$$f(t^2) = 2t - t^4$$

$$t = \frac{\pi}{2} \Rightarrow f\left(\frac{\pi^2}{4}\right) = \frac{2\pi}{2} - \frac{\pi^4}{16}$$

$$= \pi - \frac{\pi^4}{16} = \pi\left(1 - \frac{\pi^3}{16}\right)$$

2. Eight persons are to be transported from city A to city B in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is:

(1) 3360

(2) 1680

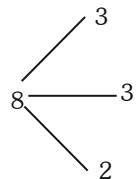
(3) 560

(4) 1120

Official Ans. by NTA (1)

Allen Ans. (2)

Sol.



$$\text{Ways} = \frac{8!}{3!3!2!2!} \times 3!$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4}{4}$$

$$= 56 \times 30$$

$$= 1680$$

3. For, $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if

$$\int \left(\left(\frac{x}{e} \right)^{2x} + \left(\frac{e}{x} \right)^{2x} \right) \log_e x dx = \frac{1}{\alpha} \left(\frac{x}{e} \right)^{\beta x} - \frac{1}{\gamma} \left(\frac{e}{x} \right)^{\delta x} + C,$$

Where $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ and C is constant of integration,

then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to:

(1) 1

(2) -4

(3) -8

(4) 4

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $(x = e^{\ln x})$

$$\int \left(\left(\frac{x}{e} \right)^{2x} + \left(\frac{e}{x} \right)^{2x} \right) \log_e x dx = \int \left[e^{2(x \ln x - x)} + e^{-2(x \ln x - x)} \right] \ln x dx$$

$$x \ln x - x = t$$

$$\ln x \cdot dx = dt$$

$$\int (e^{2t} + e^{-2t}) dt$$

$$\frac{e^{2t}}{2} - \frac{e^{-2t}}{2} + C$$

$$= \frac{1}{2} \left(\frac{x}{e} \right)^{2x} - \frac{1}{2} \left(\frac{e}{x} \right)^{2x} + C$$

$$\alpha = \beta = \gamma = \delta = 2$$

$$\alpha + 2\beta + 3\gamma - 4\delta = 4$$

4. Let the image of the point $P(1, 2, 6)$ in the plane passing through the points $A(1, 2, 0)$, $B(1, 4, 1)$ and $C(0, 5, 1)$ be $Q(\alpha, \beta, \gamma)$. Then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to :

- (1) 65
- (2) 70
- (3) 76
- (4) 62

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Equation of plane $A(x - 1) + B(y - 2) + C(z - 0) = 0$

$$\text{Put } (1, 4, 1) \Rightarrow 2B + C = 0$$

$$\text{Put } (0, 5, 1) \Rightarrow -A + 3B + C = 0$$

$$\text{Sub : } B - A = 0 \Rightarrow A = B, C = -2B$$

$$1(x - 1) + 1(y - 2) - 2(z - 0) = 0$$

$$x + y - 2z - 3 = 0$$

Image is (α, β, γ) pt $\equiv (1, 2, 6)$

$$\frac{\alpha - 1}{1} = \frac{\beta - 2}{1} = \frac{\gamma - 6}{-2} = \frac{-2(1+2-12-3)}{6}$$

$$\frac{\alpha - 1}{1} = \frac{\beta - 2}{1} = \frac{\gamma - 6}{-2} = 4$$

$$\alpha = 5, \beta = 6, \gamma = -2 \Rightarrow \alpha^2 + \beta^2 + \gamma^2$$

$$= 25 + 36 + 4 = 65$$

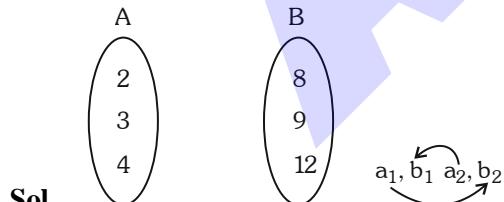
5. Let $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Then the number of elements in the relation

$R = \{(a_1, b_1), (a_2, b_2)\} \in (A \times B, A \times B) : a_1 \text{ divides } b_2 \text{ and } a_2 \text{ divides } b_1\}$ is :

- (1) 36
- (2) 12
- (3) 18
- (4) 24

Official Ans. by NTA (1)

Allen Ans. (1)



Sol.

a_1 divides b_2

Each element has 2 choices

$$\Rightarrow 3 \times 2 = 6$$

a_2 divides b_1

Each element has 2 choices

$$\Rightarrow 3 \times 2 = 6$$

$$\text{Total} = 6 \times 6 = 36$$

6. If $A = \frac{1}{5!6!7!} \begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|\text{adj}(\text{adj}(2A))|$ is

equal to :

- (1) 2^8
- (2) 2^{12}
- (3) 2^{20}
- (4) 2^{16}

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $|\text{adj}(\text{adj}(2A))| = |2A|^{(n-1)^2}$

$$= |2A|^4$$

$$= (2^3 |A|)^4$$

$$= 2^{12} |A|^4 \Rightarrow 2^{16}$$

$$|A| = \frac{1}{5!6!7!} \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$$R_3 \rightarrow R_3 \rightarrow R_2$$

$$R_2 \rightarrow R_2 \rightarrow R_1$$

$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$

7. Let A be the point $(1, 2)$ and B be any point on the curve $x^2 + y^2 = 16$. If the centre of the locus of the point P , which divides the line segment AB in the ratio $3 : 2$ is the point $C(\alpha, \beta)$, then the length of the line segment AC is

$$(1) \frac{6\sqrt{5}}{5} \quad (2) \frac{4\sqrt{5}}{5}$$

$$(3) \frac{2\sqrt{5}}{5} \quad (4) \frac{3\sqrt{5}}{5}$$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. A(1, 2) P(h,k) B(4cosθ, 4sinθ)

$$\frac{12\cos\theta + 2}{5} = h \Rightarrow 12\cos\theta = 5h - 2$$

$$\frac{12\sin\theta + 4}{5} = k \Rightarrow 12\sin\theta = 5k - 4$$

Sq & add :

$$144 = (5h - 2)^2 + (5k - 4)^2$$

$$\left(x - \frac{2}{5}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{144}{25}$$

$$\text{Centre} \equiv \left(\frac{2}{5}, \frac{4}{5}\right) \equiv (\alpha, \beta)$$

$$AC = \sqrt{\left(1 - \frac{2}{5}\right)^2 + \left(2 - \frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{36}{25}} = \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$$

- 8.** Let a die be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to :

- (1) 30
- (2) 90
- (3) 15
- (4) 60

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $P(\text{odd number 7 times}) = P(\text{odd number 9 times})$

$${}^nC_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{n-7} = {}^nC_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{n-9}$$

$${}^nC_7 = {}^nC_9$$

$$\Rightarrow n = 16$$

Required

$$P = {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$$

$$= \frac{16 \cdot 15}{2} \times \frac{1}{2^{16}} = \frac{15}{2^{13}}$$

$$\Rightarrow \frac{60}{2^{15}} \Rightarrow k = 60$$

- 9.** Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0, x \in (0, 1)$.

If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha, 1)$, then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{1}{\alpha}\right) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to :

$$(1) \frac{3\pi}{2}$$

$$(2) \pi$$

$$(3) \frac{5\pi}{4}$$

$$(4) \frac{3\pi}{4}$$



Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $g(x) = f(x) + f(1-x) \& f'(x) > 0, x \in (0, 1)$

$$g'(x) = f'(x) - f'(1-x) = 0$$

$$\Rightarrow f'(x) = f'(1-x)$$

$$x = 1 - x$$

$$x = \frac{1}{2}$$

$$g'(x) = 0$$

$$\text{at } x = \frac{1}{2}$$

$$g''(x) = f''(x) + f''(1-x) > 0$$

g is concave up

$$\text{hence } \alpha = \frac{1}{2}$$

$$\tan^{-1} 2\alpha + \tan^{-1} \frac{1}{\alpha} + \tan^{-1} \frac{\alpha+1}{\alpha}$$

$$\Rightarrow \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$$

- 10.** Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle.

$$(1) \frac{\pi}{4} \quad (2) \frac{\pi}{3}$$

$$(3) \frac{\pi}{12} \quad (4) \frac{\pi}{6}$$

Official Ans. by NTA (2)

Allen Ans. (2)

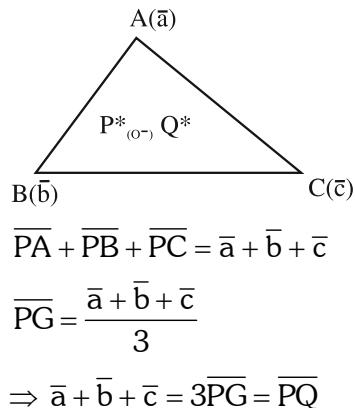
13. If the points P and Q are respectively the circumcentre and the orthocentre of a $\triangle ABC$, then $\vec{PA} + \vec{PB} + \vec{PC}$ is equal to

- (1) $2\vec{QP}$ (2) \vec{QP}
 (3) $2\vec{PQ}$ (4) \vec{PQ}

Official Ans. by NTA (4)

Allen Ans. (4)

Sol.



Ans. (4)

14. The statement $\sim[p \vee (\sim(p \wedge q))]$ is equivalent to

- (1) $(\sim(p \wedge q)) \wedge q$
 (2) $\sim(p \wedge q)$
 (3) $\sim(p \vee q)$
 (4) $(p \wedge q) \wedge (\sim p)$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\sim[p \vee (\sim(p \wedge q))]$

$$\sim p \wedge (p \wedge q)$$

15. Let $S = \left\{ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10 \right\}$ and

$$\beta = \sum_{x \in S} \tan^2 \left(\frac{x}{3} \right), \text{ then } \frac{1}{6}(\beta - 14)^2 \text{ is equal to}$$

- (1) 32
 (2) 8
 (3) 64
 (4) 16

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Let $9^{\tan^2 x} = P$

$$\frac{9}{P} + P = 10$$

$$P^2 - 10P + 9 = 0$$

$$(P - 9)(P - 1) = 0$$

$$P = 1, 9$$

$$9^{\tan^2 x} = 1, 9^{\tan^2 x} = 9$$

$$\tan^2 x = 0, \tan^2 x = 1$$

$$x = 0, \pm \frac{\pi}{4} \quad \therefore x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\beta = \tan^2(0) + \tan^2\left(\frac{\pi}{12}\right) + \tan^2\left(-\frac{\pi}{12}\right)$$

$$= 0 + 2(\tan 15^\circ)^2$$

$$2(2 - \sqrt{3})^2$$

$$2(7 - 4\sqrt{3})$$

$$\text{Then } \frac{1}{6}(14 - 8\sqrt{3} - 14)^2 = 32$$

16. If the coefficients of x and x^2 in $(1+x)^p (1-x)^q$ are 4 and -5 respectively, then $2p+3q$ is equal to

- (1) 63
 (2) 69
 (3) 66
 (4) 60

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $(1+x)^p (1-x)^q$

$$\left(1 + px + \frac{p(p-1)}{2!}x^2 + \dots\right)$$

$$\left(1 - qx + \frac{q(q-1)}{2!}x^2 - \dots\right)$$

$$p - q = 4$$

$$\frac{p(p-1)}{2} + \frac{q(q-1)}{2} - pq = -5$$

$$p^2 + q^2 - p - q - 2pq = -10$$

$$(q+4)^2 + q^2 - (q+4) - q - 2(4+q)q = -10$$

$$q^2 + 8q + 16 - q^2 - q - 4 - q - 8q - 2q^2 = -10$$

$$-2q = -22$$

$$q = 11$$

$$p = 15$$

$$2(15) + 3(11)$$

$$30 + 33 = 63$$

Sol. $\sum f_i = 62$
 $\Rightarrow 3k^2 + 16k - 12k - 64 = 0$
 $\Rightarrow k = \text{or } -\frac{16}{3} \text{ (rejected)}$

$$\mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{8 + 2(15) + 3(15) + 4(17) + 5}{62} = \frac{156}{62}$$

$$\sigma^2 = \sum f_i x_i^2 - (\sum f_i x_i)^2$$

$$= \frac{8 \times 1^2 + 15 \times 13 + 17 \times 16 + 25}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$$

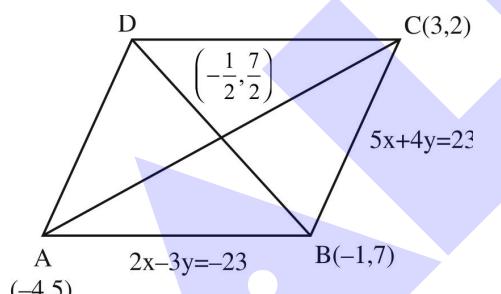
$$\sigma^2 + \mu^2 = \frac{500}{62}$$

$$[\sigma^2 + \mu^2] = 8$$

SECTION-B

21. Let the equations of two adjacent sides of a parallelogram ABCD be $2x - 3y = -23$ and $5x + 4y = 23$. If the equation of its one diagonal AC is $3x + 7y = 23$ and the distance of A from the other diagonal is d, then $50d^2$ is equal to _____.
Official Ans. by NTA (529)
Allen Ans. (529)

Sol.



A & C point will be $(-4, 5)$ & $(3, 2)$
mid point of AC will be $\left(-\frac{1}{2}, \frac{7}{2}\right)$

equation of diagonal BD is

$$y - \frac{7}{2} = \frac{\frac{7}{2}}{-\frac{1}{2}} \left(x + \frac{1}{2}\right)$$

- $\Rightarrow 7x + y = 0$
Distance of A from diagonal BD

$$= d = \frac{23}{\sqrt{50}}$$

$$\Rightarrow 50d^2 = (23)^2$$

$$50d^2 = 529$$

22. Let S be the set of values of λ , for which the system of equations
 $6\lambda x - 3y + 3z = 4\lambda^2$,
 $2x + 6\lambda y + 4z = 1$,
 $3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12 \sum_{\lambda \in S} |\lambda|$ is equal to _____.

Official Ans. by NTA (24)

Allen Ans. (24)

Sol. $\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0 \text{ (For No Solution)}$

$$2\lambda(9\lambda^2 - 4) + (3\lambda - 6) + (2 - 9\lambda) = 0$$

$$18\lambda^3 - 14\lambda - 4 = 0$$

$$(\lambda - 1)(3\lambda + 1)(3\lambda + 2) = 0$$

$$\Rightarrow \lambda = 1, -1/3, -2/3$$

For each λ , $\Delta_1 = \begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \neq 0$

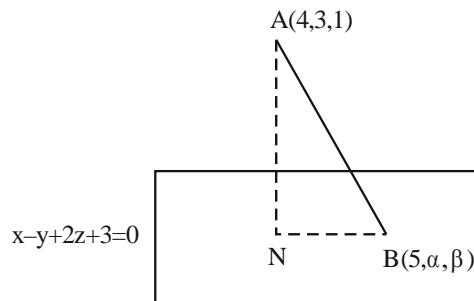
$$\text{Ans. } 12 \left(1 + \frac{1}{3} + \frac{2}{3}\right) = 24$$

23. Let the foot of perpendicular from the point A(4, 3, 1) on the plane P : $x - y + 2z + 3 = 0$ be N. If B(5, α , β), $\alpha, \beta \in \mathbb{Z}$ is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to _____.

Official Ans. by NTA (7)

Allen Ans. (7)

Sol.



$$AN = \sqrt{6}$$

$$5 - \alpha + 2\beta + 3 = 0$$

$$\Rightarrow \alpha = 8 + 2\beta \quad \dots \dots (1)$$

N is given by

$$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{2} = \frac{-(4-3+2+3)}{1+1+4}$$

$$\Rightarrow x = 3, y = 4, z = -1$$

\Rightarrow N is $(3, 4, -1)$

$$BN = \sqrt{4 + (\alpha - 4)^2 + (\beta + 1)^2}$$

$$= \sqrt{4 + (2\beta + 4)^2 + (\beta + 1)^2}$$

$$\text{Area of } \Delta ABN = \frac{1}{2} AN \times BN = 3\sqrt{2}$$

$$\Rightarrow \frac{1}{2} \times \sqrt{6} \times BN = 3\sqrt{2}$$

$$BN = 2\sqrt{3}$$

$$\Rightarrow 4 + (2\beta + 4)^2 + (\beta + 1)^2 = 12$$

$$(2\beta + 4)^2 + (\beta + 1)^2 - 8 = 0$$

$$5\beta^2 + 18\beta + 9 = 0$$

$$(5\beta + 3)(\beta + 3) = 0$$

$$\beta = -3$$

$$\Rightarrow \alpha = 2$$

$$\Rightarrow \alpha^2 + \beta^2 + \alpha\beta = 9 + 4 - 6 = 7$$

- 24.** Let quadratic curve passing through the point $(-1, 0)$ and touching the line $y = x$ at $(1, 1)$ be $y = f(x)$. Then the x-intercept of the normal to the curve at the point $(\alpha, \alpha + 1)$ in the first quadrant is _____.

Official Ans. by NTA (11)

Allen Ans. (11)

$$\text{Sol. } f(x) = (x + 1)(ax + b)$$

$$1 = 2a + 2b \quad \dots(1)$$

$$f'(x) = (ax + b) + a(x + 1)$$

$$1 = (3a + b) \quad \dots(2)$$

$$\Rightarrow b = 1/4, a = 1/4$$

$$f(x) = \frac{(x+1)^2}{4}$$

$$f'(x) = \frac{x}{2} + \frac{1}{2}$$

$$\alpha + 1 = 4$$

$$\alpha = 3$$

normal at $(3, 4)$

$$y - 4 = -\frac{1}{2}(x - 3)$$

$$y = 0$$

$$\text{Ans. 11}$$

$$\alpha + 1 = \frac{(\alpha + 1)^2}{4}, \alpha > -1$$

$$x = 8 + 3$$

- 25.** Let the tangent at any point P on a curve passing through the points $(1, 1)$ and $\left(\frac{1}{10}, 100\right)$, intersect positive x-axis and y-axis at the points A and B respectively. If $PA : PB = 1 : k$ and $y = y(x)$ is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$, $y(0) = k$, then $4y(1) - 5\log_e 3$ is equal to _____.

Official Ans. by NTA (6)

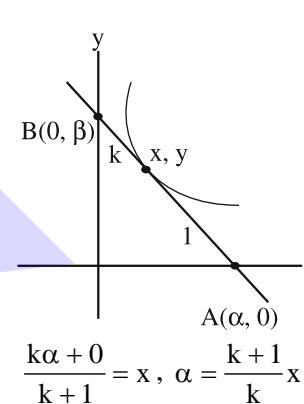
Allen Ans. (5) (answer is $4 + \ell n 3$)

- Sol.** equation of tangent at P (x, y)

$$Y - y = \frac{dy}{dx}(X - x)$$

$$Y = 0$$

$$X = \frac{-ydx}{dy} + x$$



$$\frac{k\alpha + 0}{k+1} = x, \alpha = \frac{k+1}{k} x$$

$$\frac{k+1}{k} x = -y \frac{dx}{dy} + x$$

$$x + \frac{x}{k} = -y \frac{dx}{dy} + x$$

$$x \frac{dy}{dx} + ky = 0$$

$$\frac{dy}{dx} + \frac{k}{x} y = 0$$

$$y \cdot x^k = C$$

$$C = 1$$

$$100 \cdot \left(\frac{1}{10}\right)^k = 1$$

$$K = 2$$

$$\frac{dy}{dx} = \ell n(2x+1)$$

$$y = \frac{(2x+1)}{2} (\ell n(2x+1) - 1) + c$$

$$2 = \frac{1}{2}(0 - 1) + C$$

$$C = 2 + \frac{1}{2} = \frac{5}{2}$$

$$y(1) = \frac{3}{2}(\ell n 3 - 1) + \frac{5}{2}$$

$$= \frac{3}{2} \ell n 3 + 1$$

$$4y(1) = 6\ell n 3 + 4$$

$$4y(1) - 5\ell n 3 = 4 + \ell n 3$$

26. Suppose $a_1, a_2, 2, a_3, a_4$ be in an arithmetico-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to _____.

Official Ans. by NTA (16)

Allen Ans. (16)

Sol. $\frac{(a-2d)}{4}, \frac{(a-d)}{2}, a, 2(a+d), 4(a+2d)$

a = 2

$$\left(\frac{1}{4} + \frac{1}{2} + 1 + 6\right) \times 2 + (-1 + 2 + 8)d = \frac{49}{2}$$

$$2\left(\frac{3}{4} + 7\right) + 9d = \frac{49}{2}$$

$$9d = \frac{49}{2} - \frac{62}{4} = \frac{98 - 62}{4} = 9$$

d = 1

$$\Rightarrow a_4 = 4(a + 2d) \\ = 16$$

27. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta] \cup (\gamma, \delta]$, then $|3\alpha + 10(\beta + \gamma) + 21\delta|$ is equal to _____.

Official Ans. by NTA (24)

Allen Ans. (24)

Sol. $f(x) = \sec^{-1}\frac{2x}{5x+3}$

$$\left|\frac{2x}{5x+3}\right|$$

$$\left|\frac{2x}{5x+3}\right| \geq 1 \Rightarrow |2x| \geq |5x+3|$$

$$(2x)^2 - (5x+3)^2 \geq |5x+3|^2$$

$$(7x+3)(-3x-3) \geq 0$$

$$\begin{array}{ccc} - & + & - \\ -1 & & -\frac{3}{7} \end{array}$$

\therefore domain $\left[-1, \frac{-3}{5}\right] \cup \left(\frac{-3}{5}, -\frac{3}{7}\right]$

$$\alpha = -1, \beta = \frac{-3}{5}, \gamma = \frac{-3}{5}, \delta = \frac{-3}{7}$$

$$3\alpha + 10(\beta + \gamma) + 21\delta = -3$$

$$-3 + 10\left(\frac{-6}{5}\right) + \left(\frac{-3}{7}\right) 21 = -24$$

28. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _____.

Official Ans. by NTA (26664)

Allen Ans. (26664)

Sol. **2, 1, 2, 3**

$$\begin{array}{cccccc} - & - & - & 1 & \frac{3!}{2!} = 3 \\ - & - & - & 2 & 3! = 6 \\ - & - & - & 3 & \frac{3!}{2!} = 3 \end{array}$$

Sum of digits of unit place = $3 \times 1 + 6 \times 2 + 3 \times 3 = 24$

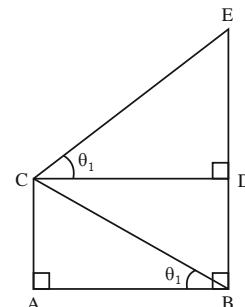
\therefore required sum

$$= 24 \times 1000 + 24 \times 100 + 24 \times 10 + 24 \times 1 \\ = 24 \times 1111$$

Ans ; 26664

29. In the figure, $\theta_1 + \theta_2 = \frac{\pi}{2}$ and $\sqrt{3}(BE) = 4(AB)$.

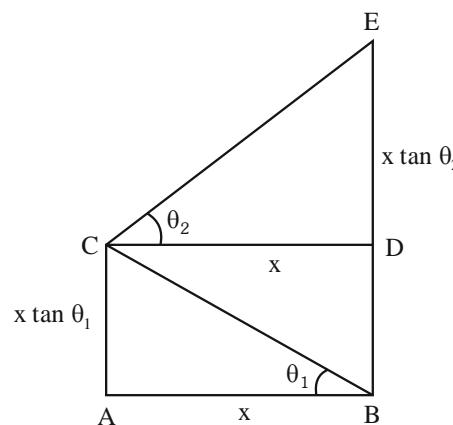
If the area of $\triangle CAB$ is $2\sqrt{3} - 3$ unit², when $\frac{\theta_2}{\theta_1}$ is the largest, then the perimeter (in unit) of $\triangle CED$ is equal to _____.



Official Ans. by NTA (6)

Allen Ans. (6)

Sol.



$$\sqrt{3} BE = 4 AB$$

$$Ar(\Delta CAB) = 2\sqrt{3} - 3$$

$$\frac{1}{2}x^2 \tan \theta_1 = 2\sqrt{3} - 3$$

$$BE = BD + DE$$

$$= x (\tan \theta_1 + \tan \theta_2)$$

$$BE = AB (\tan \theta_1 + \cot \theta_1)$$

$$\frac{4}{\sqrt{3}} \tan \theta_1 + \cot \theta_1 \Rightarrow \tan \theta_1 = \sqrt{3}, \frac{1}{\sqrt{3}}$$

$$\theta_1 = \frac{\pi}{6}$$

$$\theta_2 = \frac{\pi}{3}$$

$$\theta_1 = \frac{\pi}{3}$$

$$\theta_2 = \frac{\pi}{6}$$

$$\text{as } \frac{\theta_2}{\theta_1} \text{ is largest } \therefore \theta_1 = \frac{\pi}{6}, \theta_2 = \frac{\pi}{3}$$

$$\therefore x^2 = \frac{(2\sqrt{3} - 3) \times 2}{\tan \theta_1} = \frac{\sqrt{3}(2 - \sqrt{3}) \times 2}{\tan \frac{\pi}{6}}$$

$$x^2 = 12 - 6\sqrt{3} = (3 - \sqrt{3})^2$$

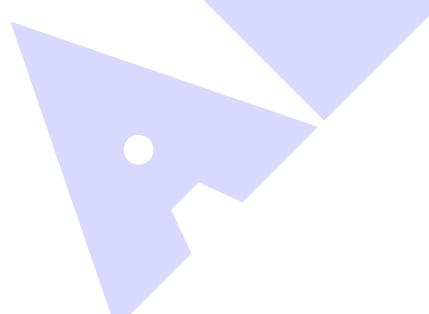
$$x = 3 - \sqrt{3}$$

Perimeter of ΔCED

$$= CD + DE + CE$$

$$= 3\sqrt{3} + (3 - \sqrt{3})\sqrt{3} + (3 - \sqrt{3}) \times 2 = 6$$

Ans : 6

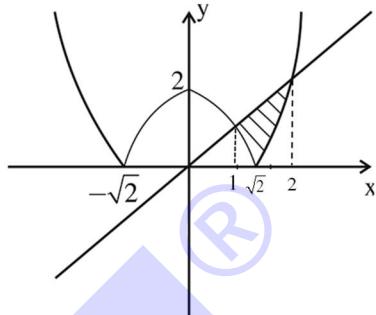


30. If the area of the region $\{(x,y) : |x^2 - 2| \leq y \leq x\}$ is A, then $6A + 16\sqrt{2}$ is equal to _____.

Official Ans. by NTA (27)

Allen Ans. (27)

Sol. $|x^2 - 2| \leq y \leq x$



$$\begin{aligned} A &= \int_1^{\sqrt{2}} (x - (2 - x^2)) dx + \int_{\sqrt{2}}^2 (x - (x^2 - 2)) dx \\ &= \left[1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3} \right] - \left[\frac{1}{2} - 2 + \frac{1}{3} \right] + \left[2 - \frac{8}{3} + 4 \right] - \left[1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2} \right] \\ &= -4\sqrt{2} + \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} = \frac{-8\sqrt{2}}{3} + \frac{9}{2} \end{aligned}$$

$$6A = -16\sqrt{2} + 27 \therefore 6A + 16\sqrt{2} = 27$$

Ans : 27

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